Aalto-yliopisto
Perustieteiden korkeakoulu

Accuracy of approximate operations on fuzzy numbers

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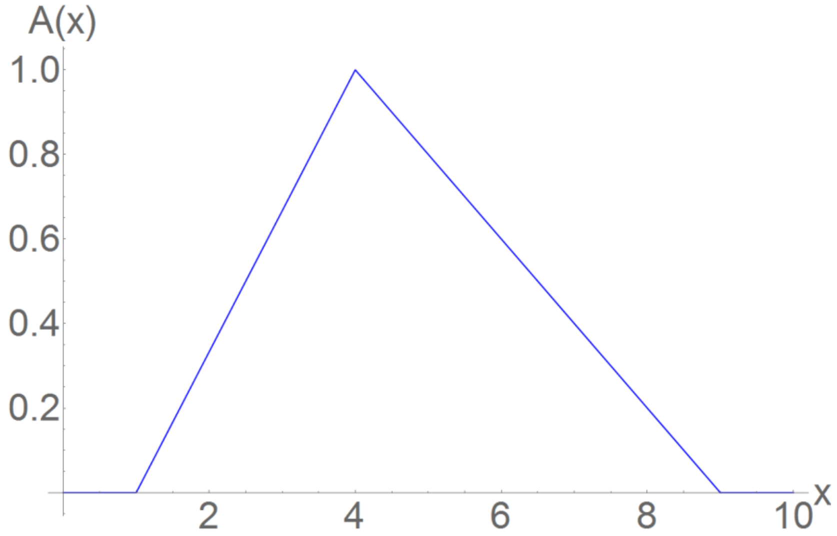
Työn saa tallentaa ja julkistaa Aalto-yliopiston avoimilla verkkosivuilla. Muilta osin kaikki oikeudet pidätetään.

Systeemianalyysin laboratorio

Fuzzy number

- A fuzzy set of type $A: \mathbb{R} \longrightarrow [0,1]$
- Continuous membership function $A(x)$



Background

- Fuzzy multiplication does not preserve linearity.
- In literature it is often approximated to preserve linearity as to save computational power.
- Error caused by this approximation is typically assumed to be negligible.
- This may cause errors in ranking fuzzy numbers.

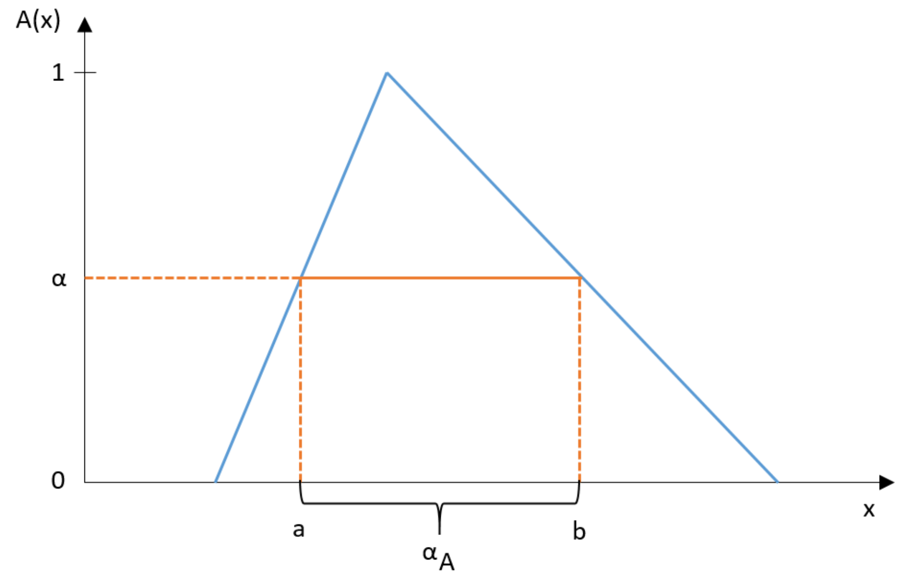
Objectives of the thesis

- Determine significance of the approximated preserved linearity for multiplication.
- Further analyze scenarios where error is significant.

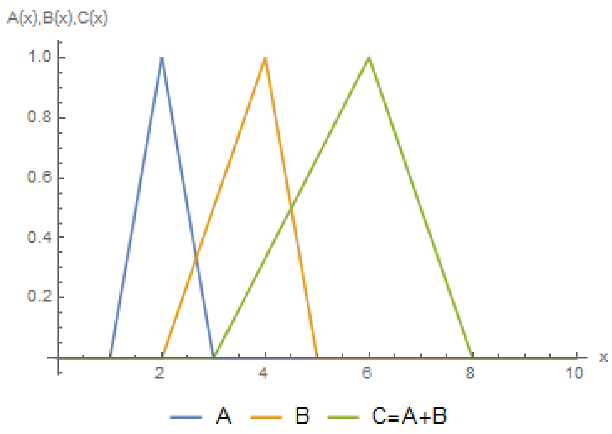
Alpha cuts

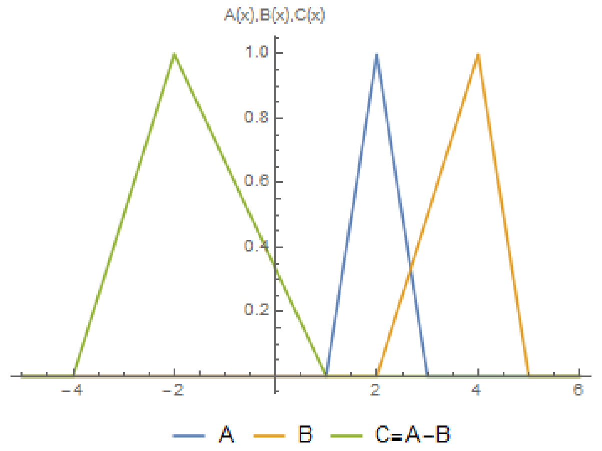
- Alpha cut is a crisp set that contains all members of the fuzzy set whose grade of membership is greater than α.
- All fuzzy sets can fully and uniquely be described by their alpha cuts.

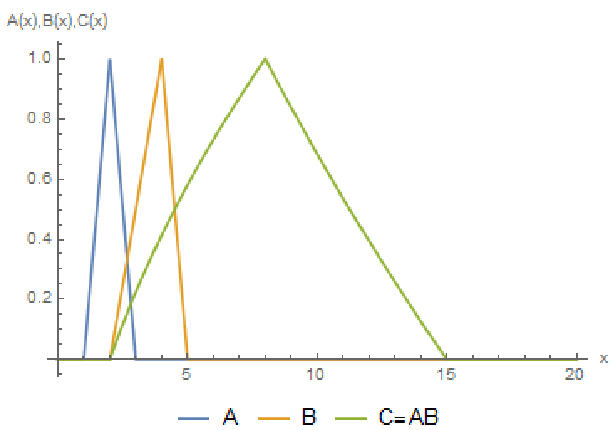
$$^{\alpha}A = \{x | A(x) \geq \alpha\}$$

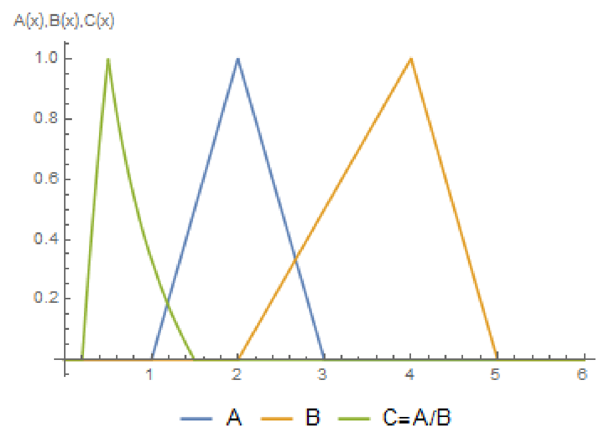


Fuzzy arithmetic

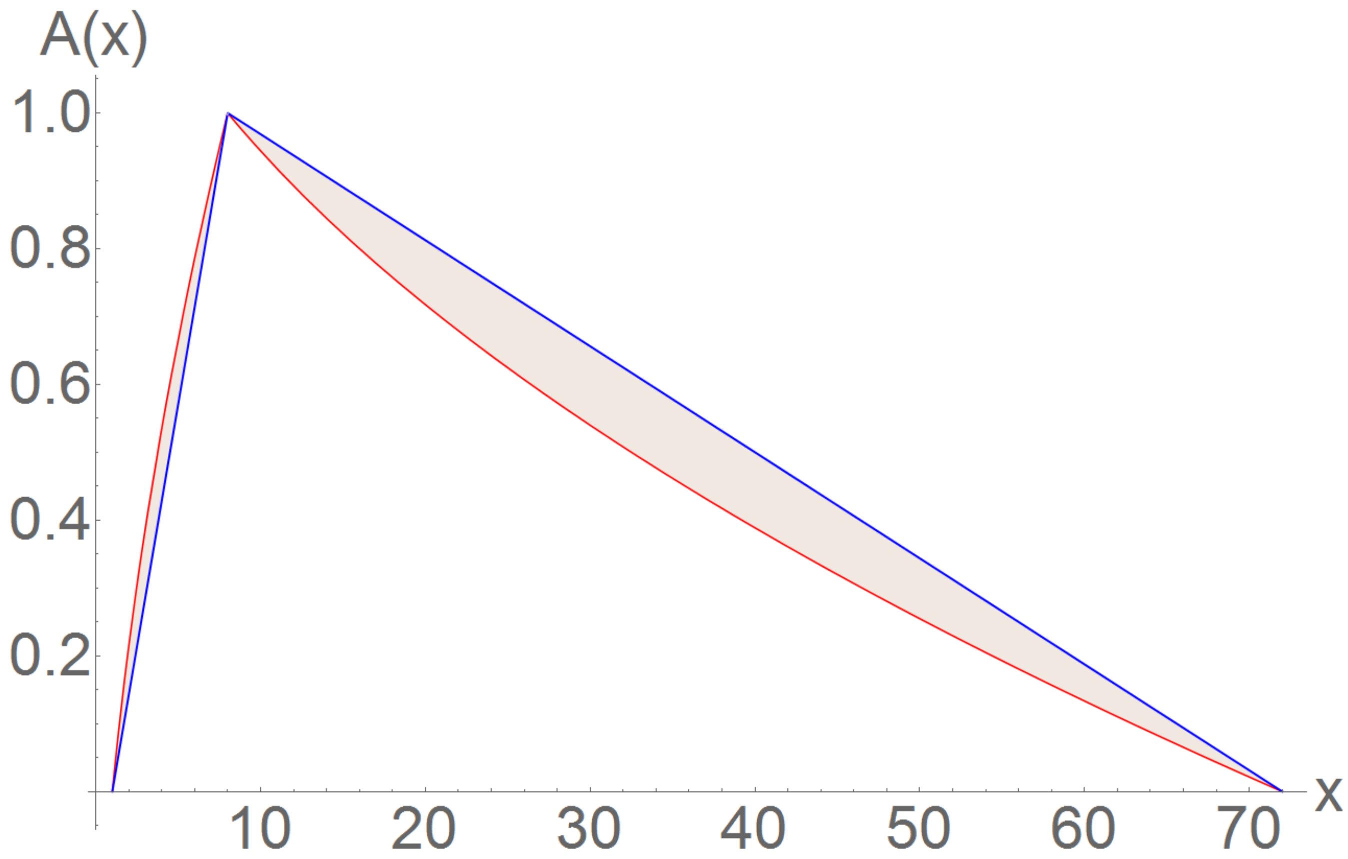








Error in linear approximation of the product



Ranking methods

- Used to rank Fuzzy numbers
  - First type methods used here rank fuzzy numbers to the real line
- Methods used
  - Center of Gravity

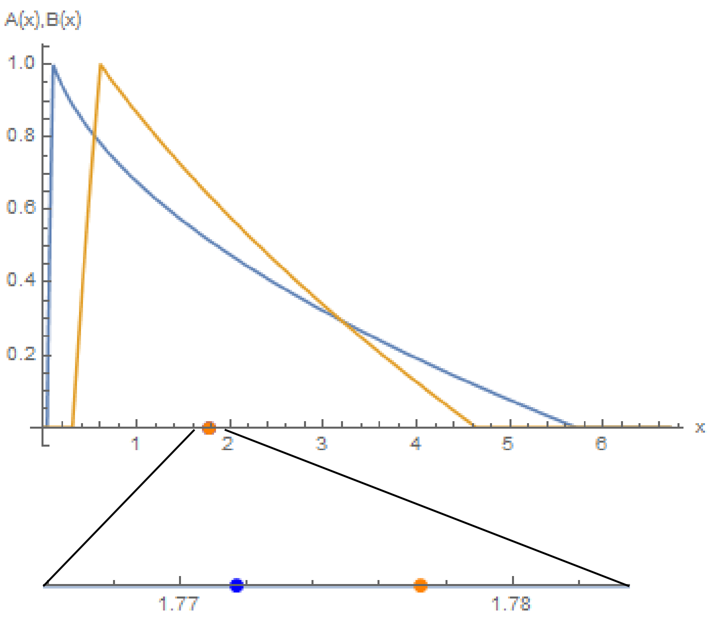
$$CoG(A) = \frac{\int_{-\infty}^{\infty} xA(x)dx}{\int_{-\infty}^{\infty} A(x)dx}$$

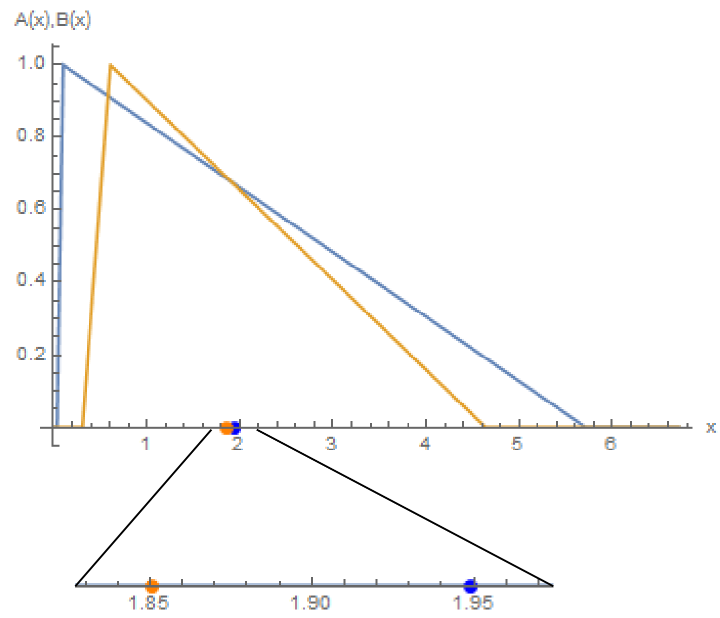
  - Possibilistic Mean

$$E_p(A) = \int_0^1 \alpha(a_\alpha^- + a_\alpha^+)d\alpha$$

Error in ranking

- Linearity approximation flips center of gravity ranking in this example



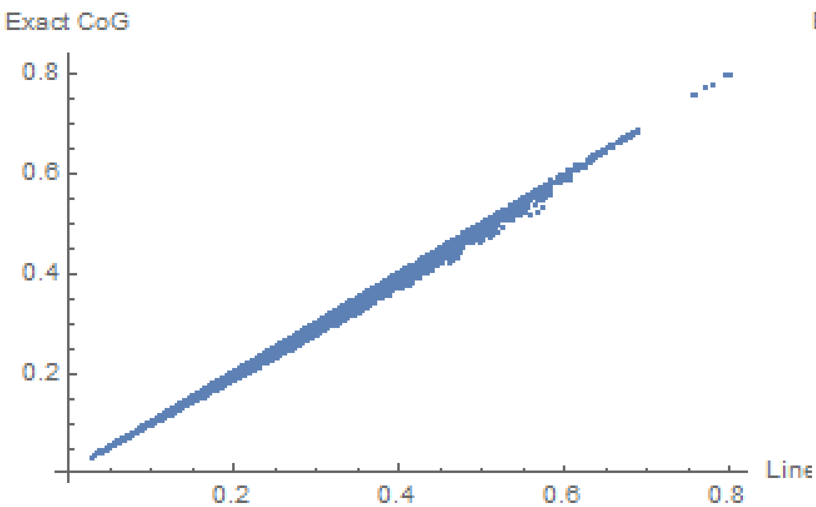


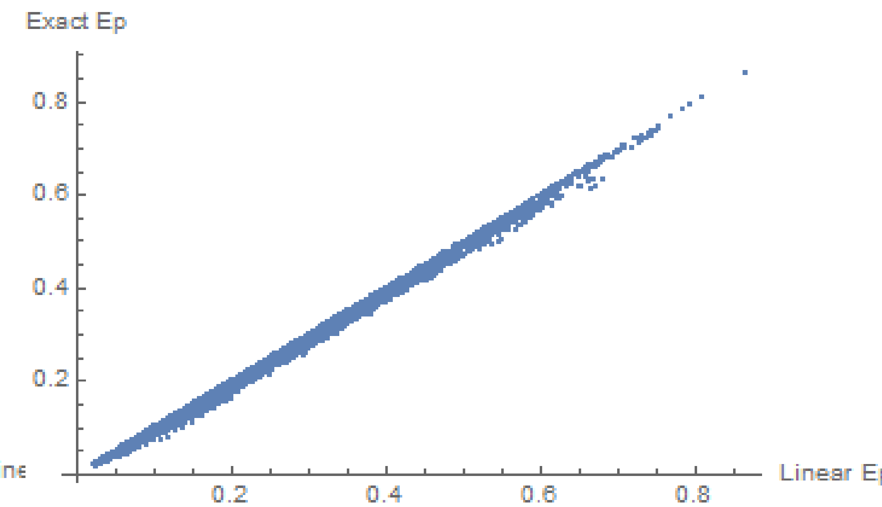
Numerical study

- Using Mathematica
  - Create 100 random triangular fuzzy numbers in range [0,1]
  - Execute each pairwise multiplication exactly to gain a fuzzy number
  - Rank the fuzzy number and its linear approximation using center of gravity and possibilistic mean methods
  - Compare the exact and linearity assuming rankings using Spearman’s rank coefficient and Kendall’s rank coefficient
    - Spearman – monotonicity
    - Kendall – pairwise agreement

Results

1st ranking method	2nd ranking method	Spearman ρ	Kendall τ	Pairwise agreement
Exact CoG	Linear CoG	0.9986	0.9680	0.9840
Exact E_p	Linear E_p	0.9985	0.9661	0.9830





Conclusions

- Pairwise agreement between exact and linearity assuming ranking over 98% for both methods
- Difference in exact ranking typically small when linearity assumption flips ranking
  - Significance of error thus small if ranking method suitably chosen
- Error builds up in consequent operations
  - Approximating preserved linearity causes significant error if applied more than once
- Using the approximation is a compromise between accuracy and computational overhead
- Results for multiplication also apply to division