Multistage investment under two sources of uncertainty – a real options approach

Lauri Kauppinen
4.11.2013

Instructor: Afzal Siddiqui (University College London)
Supervisor: Ahti Salo

The document can be stored and made available to the public on the open internet pages of Aalto University. All other rights are reserved.
The goals of the study

- Examine sequential investment decisions under two explicit sources of uncertainty
  - The model is based on the one-factor real options model of Majd & Pindyck [1987]
  - Also, McDonald & Siegel [1986] studied a two-factor model where the investment program can be finished instantaneously
    - Examples: R&D projects, new technology adoption
- Particularly study how the inclusion of the second stochastic variable affects the optimal investment policy
- Solve the model numerically
The model

- The sources of uncertainty are modeled by two stochastic variables, i.e., the discounted cash inflows and outflows of the finished project
  - We will denote these by $V$ and $C$, respectively
  - These are assumed to follow uncorrelated geometric Brownian motions with parameters $(\alpha_{V}, \sigma_{V})$ and $(\alpha_{C}, \sigma_{C})$
- The required rate of return for holding the option is $\mu$
  - We implicitly assume that the investor is risk neutral as we use dynamic programming
- The maximum investment rate is denoted by $k$ and the initial investment left by $K$
- The investor can choose the investment rate continuously, and the payoff $\max(V-C, 0)$ is obtained only when $K=0$
  - How should the investor proceed with the investment program?
A few words on how the results were obtained

- We used the dynamic programming approach to real options valuation
  - The solution is a “bang-bang” one: it is optimal to either wait or invest at the maximum rate
- This combined with the assumptions led to a two-PDE free boundary problem with three independent variables, i.e., $V$, $C$ and $K$
  - McDonald & Siegel [1986] provided an analytical solution that is linear homogenous in $V$ and $C$
    - However, this is not the case for the problem here because of the time-to-build issue
- The problem was then solved using an explicit finite difference method
  - The option value function $F(V,C,K)$ and the investment threshold $V^*(C,K)$
  - Then, the effects of the parameters on the results were studied using the method of comparative statics
The base case
($\alpha_V=\alpha_C=0.04$, $\sigma_V=\sigma_C=0.14$, $\mu=0.08$, $k=1$)
The base case
($\alpha_v=\alpha_c=0.04$, $\sigma_v=\sigma_c=0.14$, $\mu=0.08$, $k=1$)
The base case
($\alpha_V = \alpha_C = 0.04, \sigma_V = \sigma_C = 0.14, \mu = 0.08, k = 1$)
Sensitivity with respect to $\alpha_C$
(The other parameters are the same as in the base case)
The explanation

- When $K << 1$, a decrease (increase) in $\alpha_C$ increases (decreases) the incentives of waiting [McDonald & Siegel, 1986]
  - The threshold shifts up (down)
- At larger values of $K$, the optimal investment policy can be explained by the principle of dynamic programming
  - The investor knows the optimal investment policy for smaller values of $K$
  - Both the payoff and the initial investment outflows are discounted
  - It is optimal to invest so that the investment program can be completed without pauses in most cases
- Shouldn’t this imply that the effect of $\alpha_C$ on the investment threshold is amplified when $k$ is smaller and, thus, the minimum construction time is longer?
Sensitivity with respect to $\alpha_c$, when $k=0.5$
(The other parameters are the same as in the base case)
Sensitivity with respect to other parameters

- The logic behind the effect of $a_V$ on the results is the same as above
  - However, $V^*(C,K)$ grows without bounds as $a_V \rightarrow \mu$
- As $\mu$ represent the cost of waiting, an increase (decrease) in its value shifts the investment threshold down (up)
- An increase (decrease) in either of the volatilities increases (decreases) the value of waiting and therefore shifts the investment threshold up (down)
- If the increments of the stochastic variables were positively (negatively) correlated, the volatility of the process that the payoff follows would decrease (increase) shifting the investment threshold down (up)
Summary

• The investor’s problem was solved numerically yielding both the option value and the investment threshold

• Comparative statics was used to analyze the impacts of the different parameters on the optimal investment policy
  – The effects of the drift terms were explained in the framework of dynamic programming

• The model is general and can be applied in situations that meet the underlying assumptions by modifying the boundary conditions
References

• *Time to build, option value, and investment decisions*, Majd & Pindyck, 1994
• *The value of waiting to invest*, McDonald & Siegel, 1986