

Optimization and Equilibrium Modeling for Energy Markets

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Outline

1. Three Broad Research Areas
2. Mathematical Modeling Focus
3. Highlighted Research Area 1: Demand Response in Power Markets using Monte Carlo Simulation and Dynamic Programming
4. Highlighted Research Area 2: Worldwide Natural Gas Markets and Liquefied Natural Gas (LNG) using a Large-Scale, Nash-Cournot Market Equilibrium Model
5. Highlighted Research Area 3: Power Market Investments and Operations using Two-Level Optimization/Equilibrium Modeling



Three Broad Research Areas



Three Broad Research Areas

Energy System Modeling- Business Case Modeling (Private Sector)

Energy markets/systems (power, gas), fossil fuels vs. renewables, emissions, demand response, investments, geo-politics of gas, risk analysis non-convex pricing for day-ahead, real-time power markets,

Optimization and Equilibrium (Game Theory)- Operations Research Math Focus

Nash-Cournot engineering-economic modeling; one- and two-level equilibrium models and algorithm design, stochastic, robust and deterministic approaches

Energy, the Environment and Policy: Public Sector-Based

Effects of CO₂ on gas supply chains, Regional Greenhouse Gas Initiative, redistribution of RGGI funds, energy conservation



Mathematical Modeling Focus



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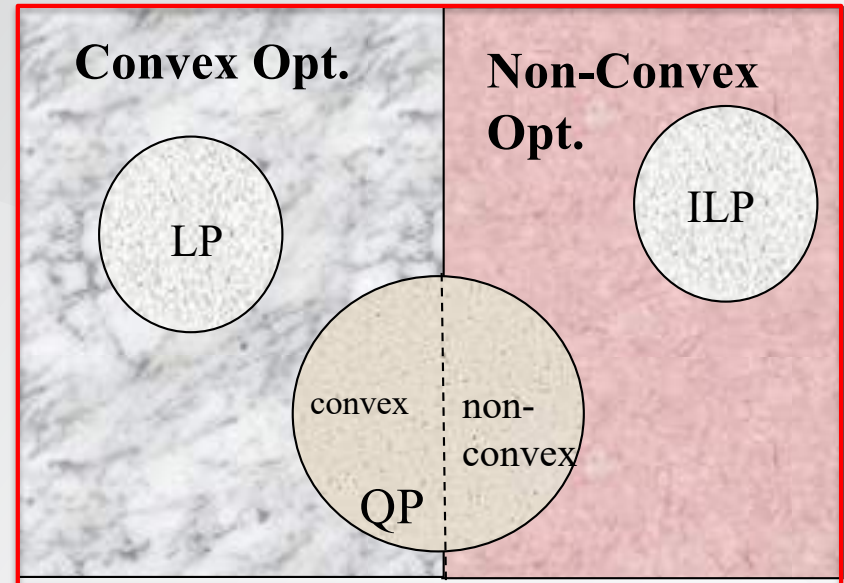
Optimization and Equilibrium Modeling: Engineering-Economic System Focus

General Form of an Optimization Problem

$\min f(x)$ Objective Function

s.t. Feasible Region

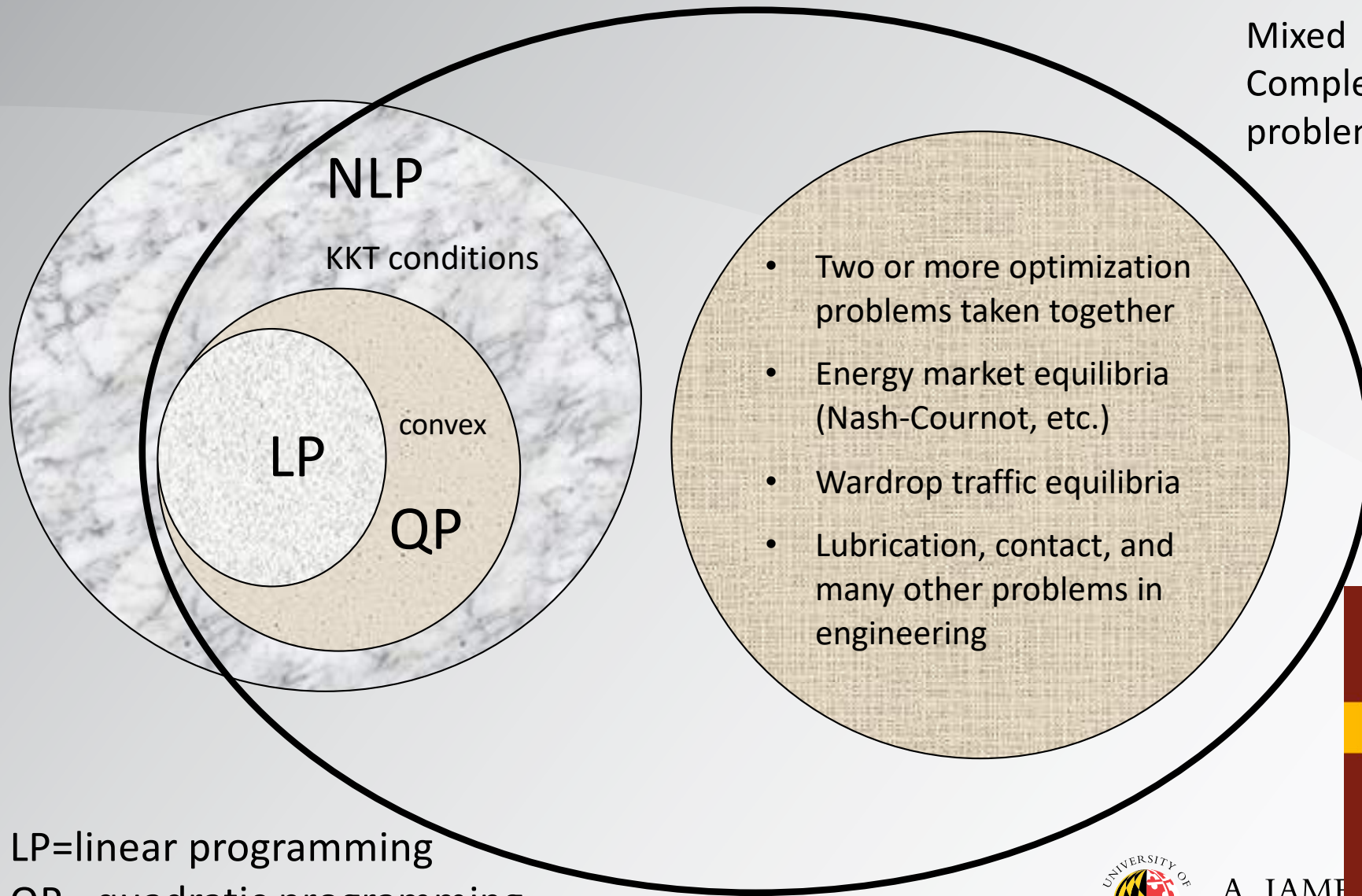
$$\left(\begin{array}{l} g_i(x) \leq 0, i = 1, 2, \dots, m \\ h_j(x) = 0, j = 1, 2, \dots, p \end{array} \right) = S$$



- Many engineering problems have either f a non-convex function or S a non-convex usually making the problem much harder to solve, examples:
 - Unit commitment in power (binary-constrained problem)
 - Alternating current optimal power flow in power
 - Weymouth equation (pressure-flow) in natural gas



Optimization and Equilibrium Modeling: Engineering-Economic System Focus



Mixed
Complementarity
problems (MCPs)

- Two or more optimization problems taken together
- Energy market equilibria (Nash-Cournot, etc.)
- Wardrop traffic equilibria
- Lubrication, contact, and many other problems in engineering

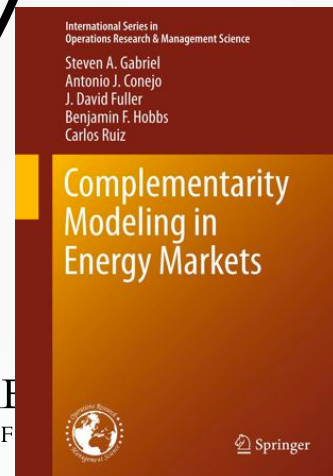
LP=linear programming

QP= quadratic programming

NLP=nonlinear programming



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Highlighted Area #1:

Dynamic Programming and Monte Carlo Simulation
for Demand Response in Power Markets

State of Maryland/Whisker Labs
Maryland Industrial Partnership Program



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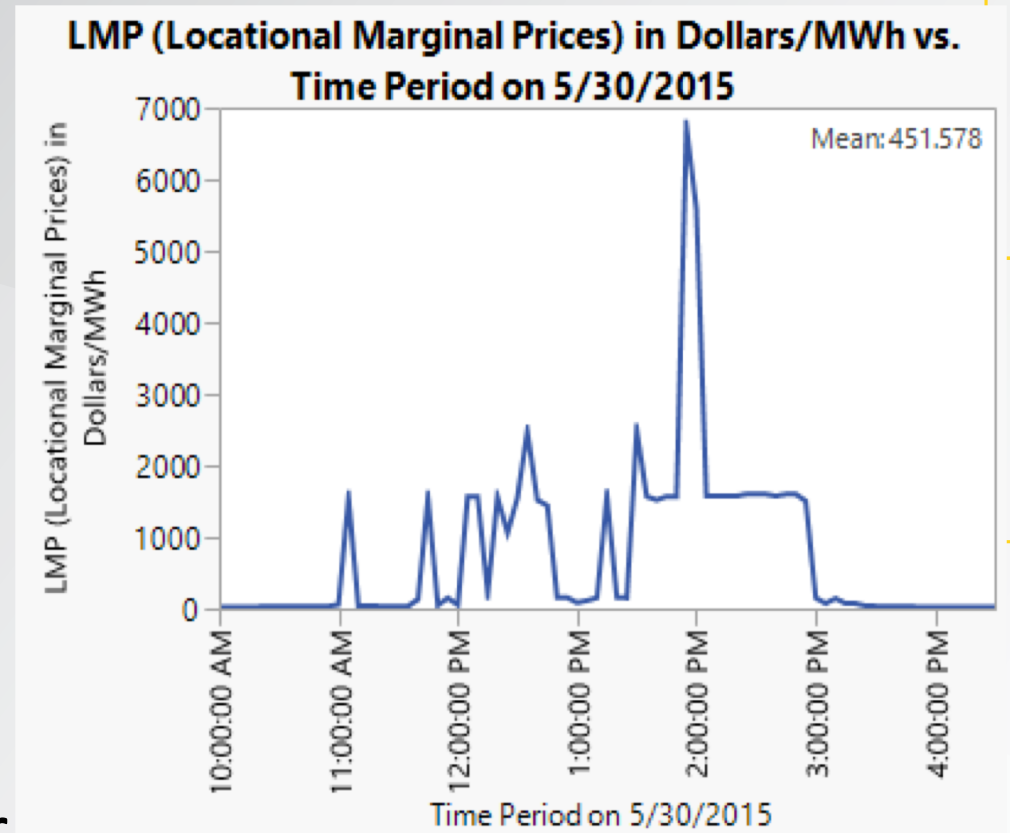
Demand Response

- Sustainability and economic motivation for demand response programs in the power sector
- The demand response residential load-shifting problem
- Two approaches from the retail electric power provider (REP) perspective with selected results
- 1. Monte Carlo Simulation
(uncertain market settlement prices, customer loads)
- 2. Deterministic & stochastic dynamic programming (optimization methodology based on Bellman's Principle of Optimality)



Economic Motivation for DR (Texas)

- Volatile market: power prices can increase by two orders of magnitude in 30 minutes
- Customers pay a constant rate, so the electric providers are fully exposed to these spikes
- Price spikes in 2011 put several retail electric power providers (REPs) out of business
- A few hours can be pivotal from a profit perspective

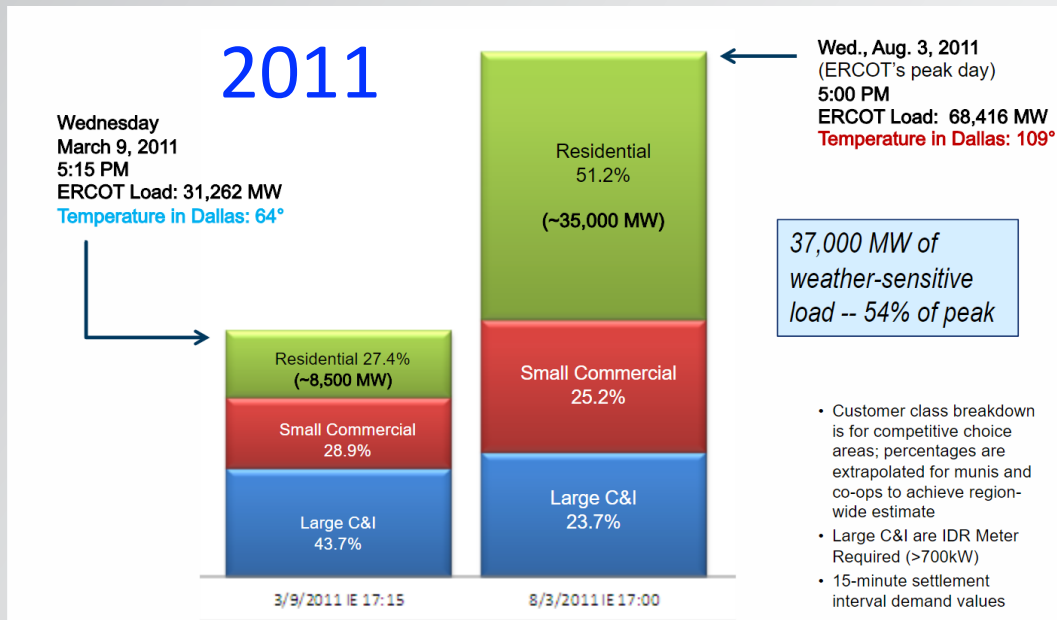
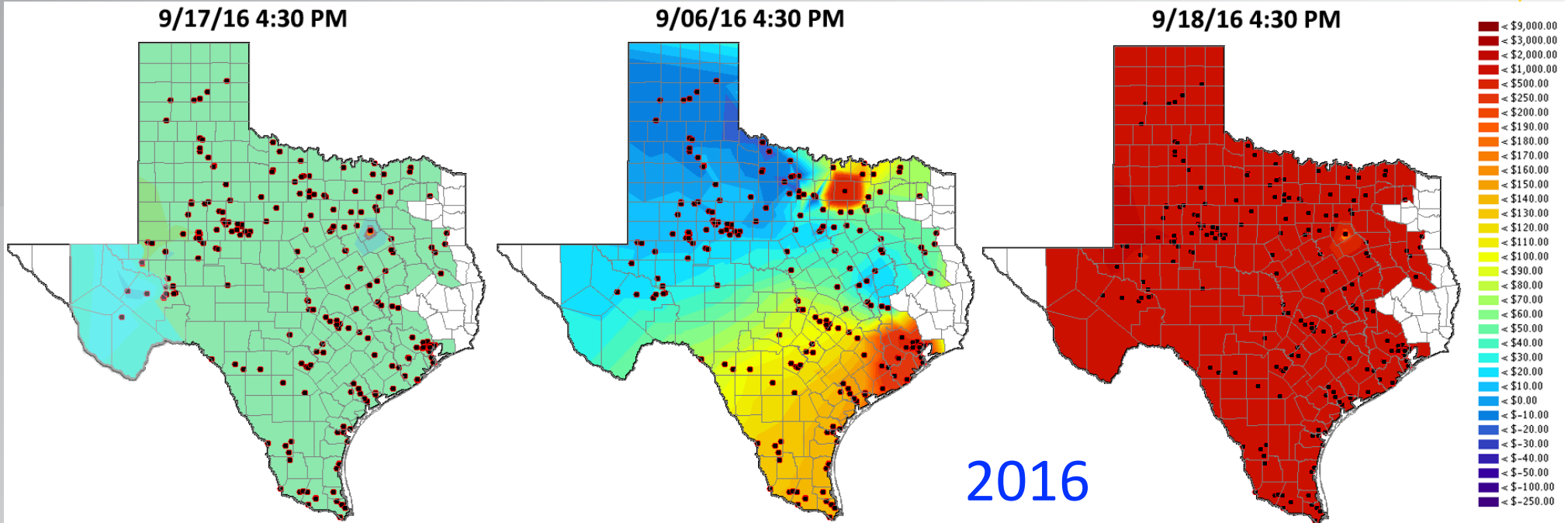


Houston, TX



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Energy Prices in Texas



- Customer class breakdown is for competitive choice areas; percentages are extrapolated for munis and co-ops to achieve region-wide estimate
- Large C&I are IDR Meter Required (>700kW)
- 15-minute settlement interval demand values

But the electricity grid is not designed for extremes!

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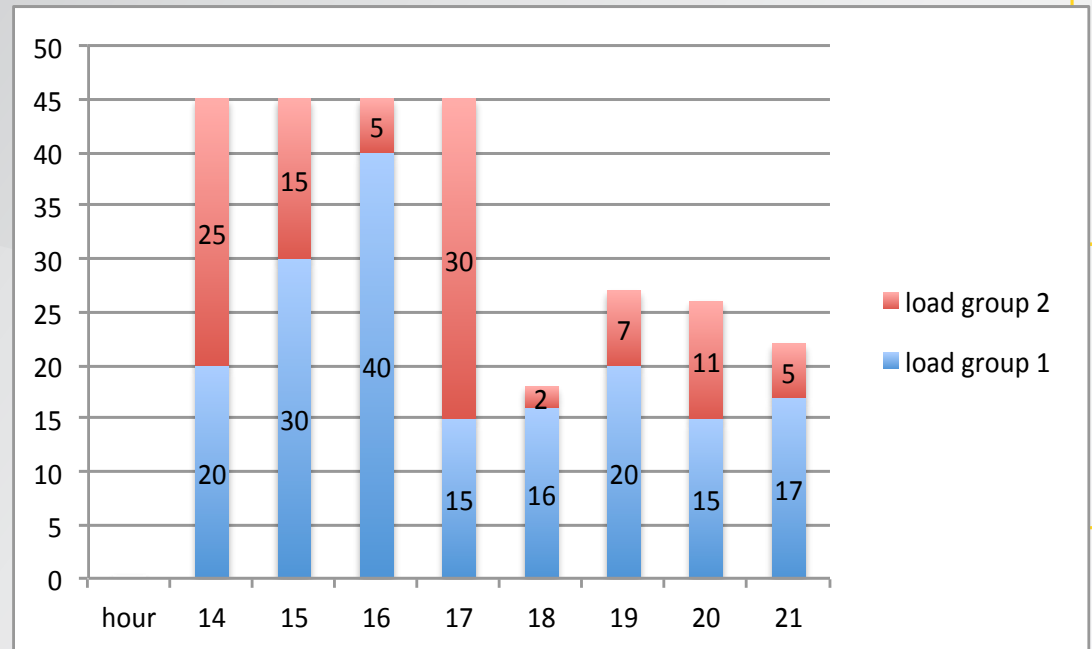
1. Monte Carlo Simulation Study (2016)

- J.R. Schaperow, S.A. Gabriel, M. Siemann, J. Crawford, 2019. "A Simulation-Based Model for Optimal Demand Response Load Shifting: Case Study for the Texas Power Market," accepted, *Journal of Energy Markets*, March 2019.



Overview of DR Modeling and Results from 2016 Monte Carlo Simulation Study

- **Central Question:** How much of each customer group's load (2 shown here) should be shifted from a current hour(s) to a contiguous hour? The shifted load will be reduced by a certain factor (thermostat setpoints, etc.)
- **Benefits:** If the load is shifted to a less expensive hour, then retail electric providers (REPs) will be able to procure the needed power for less money. Even though less load, the overall effect may be beneficial in terms of expected profit and financial risk, less need for peaking machines for the energy producer, less negative environmental impacts.

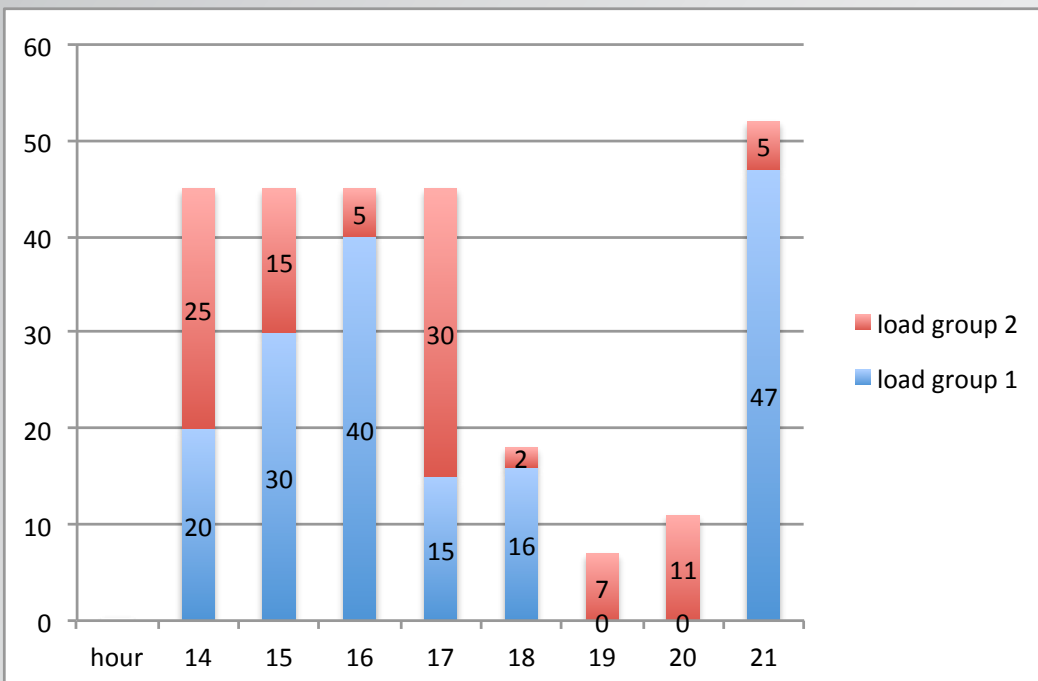
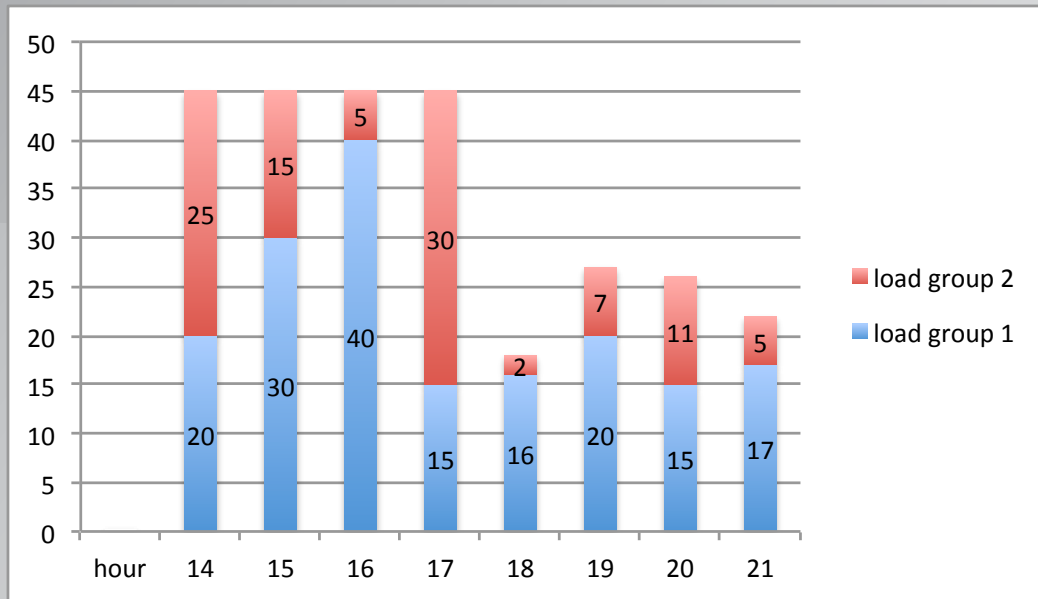


Typical day considered:

- Hours ending 14-21



Example of 2-Hour Load Shift



- Original load by group
- Group 1's load in hours ending 19 and 20 is 35 MW
- Group 1's load for hours 19 & 20 shifted to hour 21
- 35 MW reduced to 30 MW

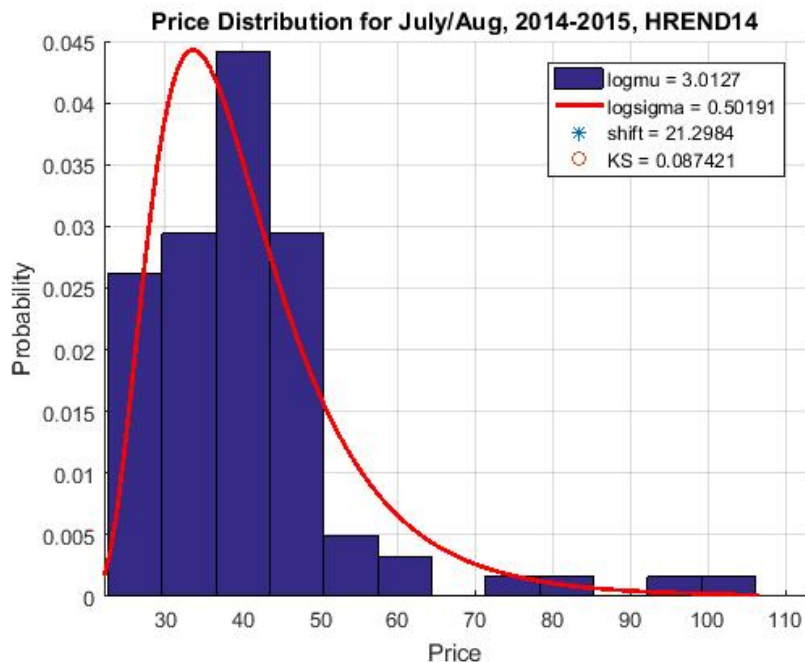
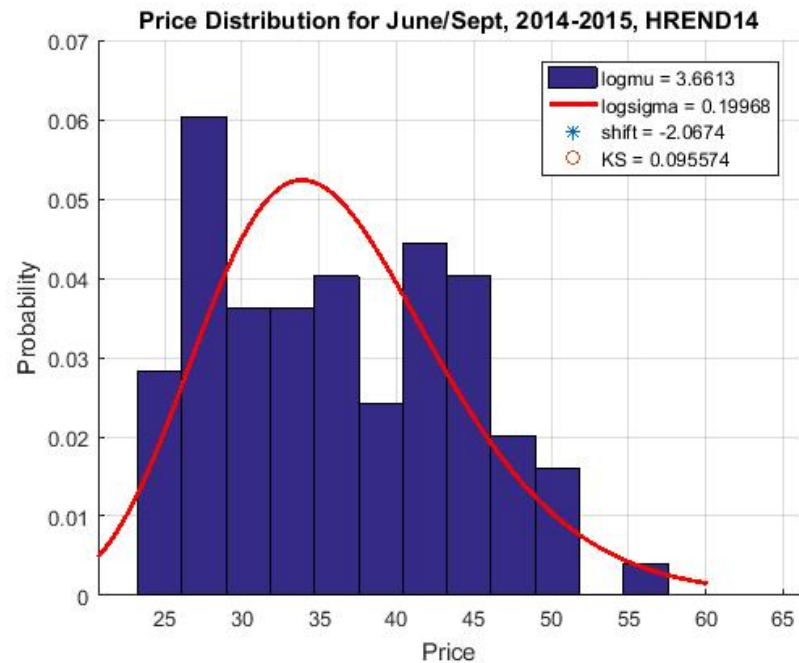


23 Possible Shifts (for each customer class)

- No shift
 - #1
- 1-hour shifts:
 - #2, 14->15
 - #3, 15->16
 - #4, 16->17
 - #5, 17->18
 - #6, 18->19
 - #7, 19->20
 - #8, 20->21
- 2-hour shifts:
 - #9, (14,15)->16
 - #10, (15,16)->17
 - #11, (16,17)->18
 - #12, (17,18)->19
 - #13, (18,19)->20
 - #14, (19,20)->21
- 3-hour shifts:
 - #15, (14,15,16)->17
 - #16, (15,16,17)->18
 - #17, (16,17,18)->19
 - #18, (17,18,19)->20
 - #19, (18,19,20)->21
- 4-hour shifts:
 - #20, (14,15,16,17)->18
 - #21, (15,16,17,18)->19
 - #22, (16,17,18,19)->20
 - #23, (17,18,19,20)->21



Settlement Price Probability Distributions



- Lognormal distribution (tail to the right)
 - Law of Proportionate Effect
 - Relatively good numerical fit
 - Different lognormal distributions for Month
 - June/September
 - July, August
 - Year (2014-2015)
 - Hour (14-21)
- Also different distributions for customer load statistically fit

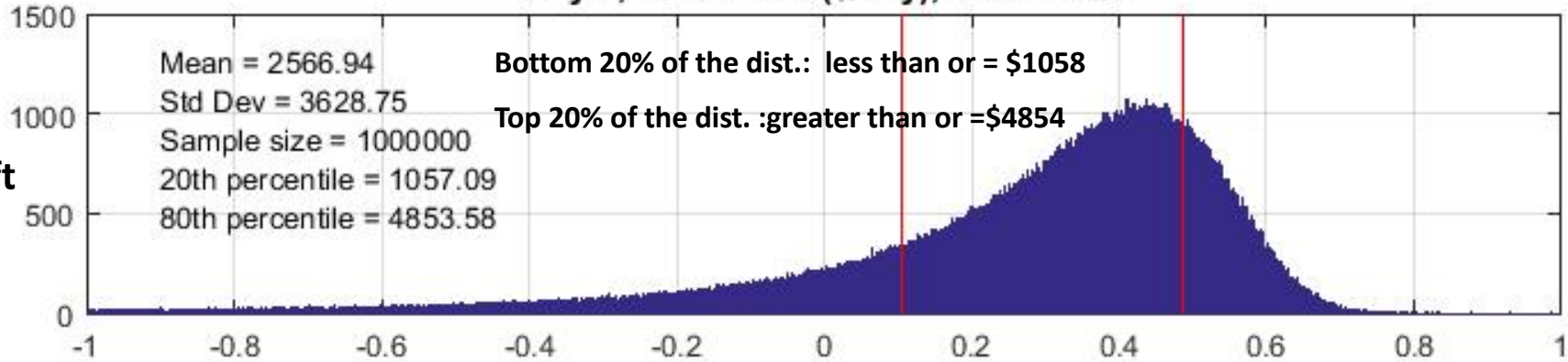


Sample Profit Distributions

Pro
fr

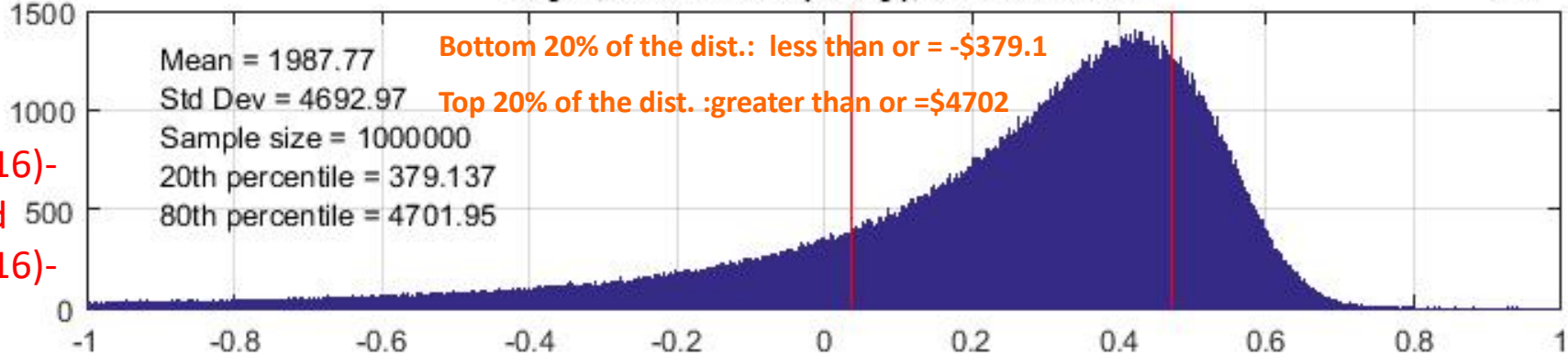
Base
Case
No shift

July 7, 2015 Profit (\$/day), DR Rule #1



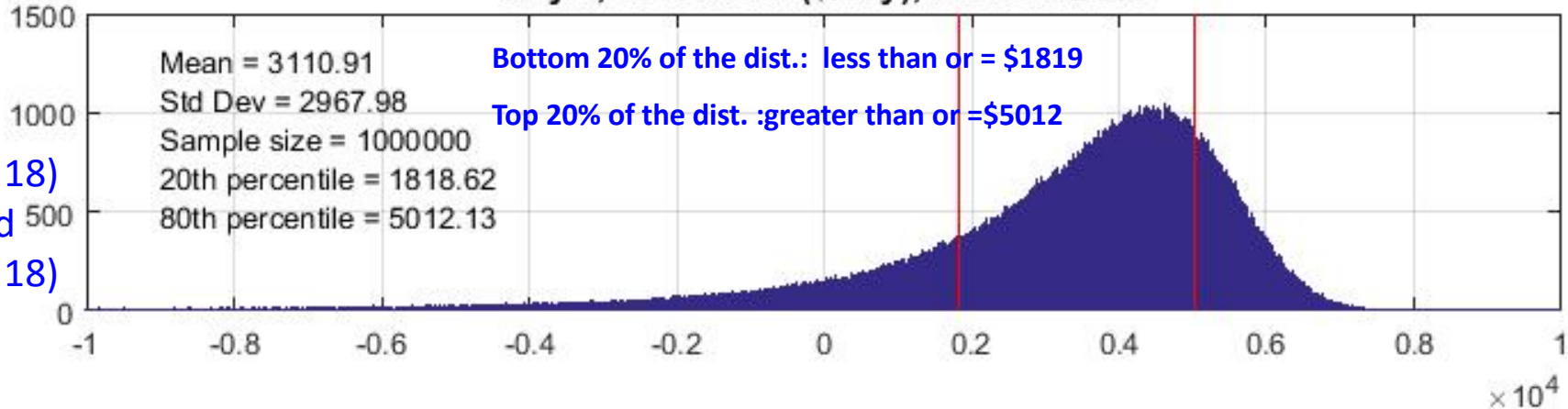
Worst
#337,
(14,15,16)-
>17 and
(14,15,16)-
>17

July 7, 2015 Profit (\$/day), DR Rule #337



Best
#385,
(16, 17, 18)
->19 and
(16, 17, 18)
->19

July 7, 2015 Profit (\$/day), DR Rule #385



2. Dynamic Programming Study (2018)

- R. L. Moglen, P. Chanpiwat, S.A. Gabriel, A. Blohm, 2018. “A Dynamic Programming Approach to Optimal Residential Demand Response Scheduling in Near Real-Time: Application for Electricity Retailers in ERCOT Power Markets,” under review, May 2018.
- A. Blohm, J. Crawford , S.A. Gabriel, 2019. “Demand Response as a Risk-Reduction Measure for Retail Electricity Providers: ERCOT Market Case Study,” under review, March, 2019.



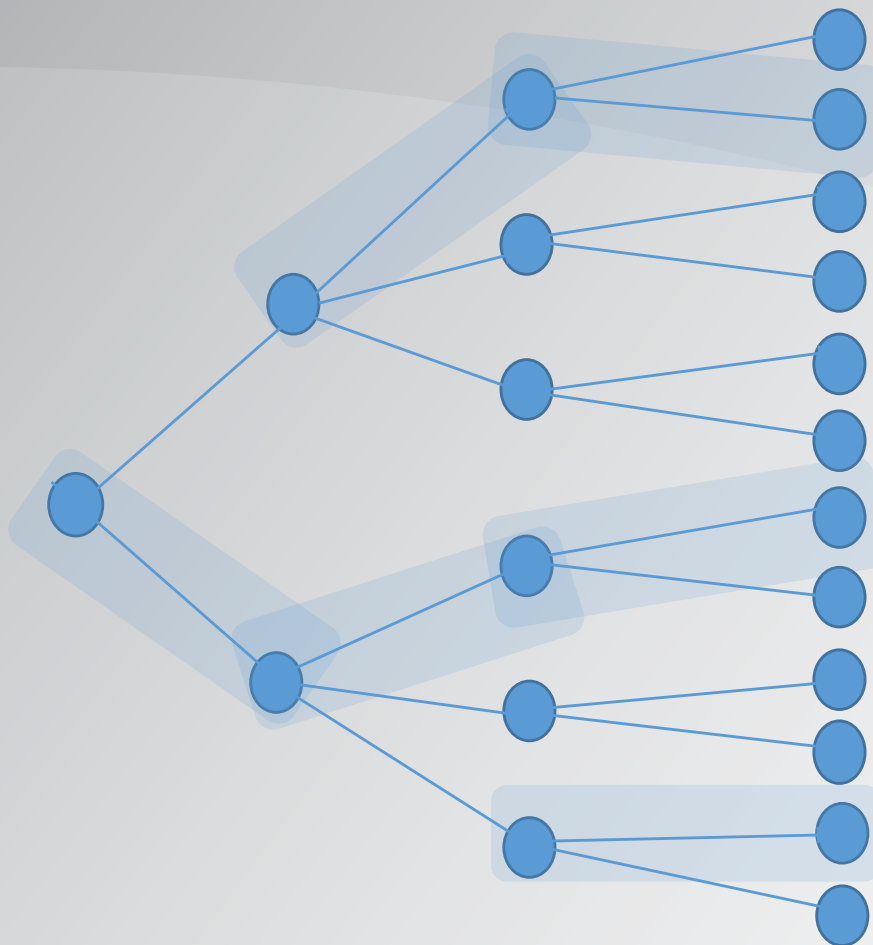
A Scheduling Problem

- Prices are volatile
- Plan optimal DR schedule by shifting load around high prices
- 17 possible schedules for 5 time periods (no DR, 1-hour, 2-hour, 3-hour, 4-hour load shifts)
- **5.9 million schedules for 24 time periods**

Time Period				
1	2	3	4	5
0	0	0	0	0
1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
1	0	1	0	0
1	0	0	1	0
0	1	0	1	0
2	0	0	0	0
2	0	1	0	0
2	0	0	1	0
0	2	0	0	0
0	0	2	0	0
1	0	2	0	0
3	0	0	0	0
0	3	0	0	0
4	0	0	0	0



Dynamic Programming



- One gets a “roadmap” of optimal decisions (based on Bellman’s Principle of Optimality)
- Simplification from exponential solving time to linear solving time w.r.t number of stages
- **Stages:** hours or half-hours (time t)
- **States:** S_t which DR events running at time t
- **Actions:** A_t 0, 1, 2,3, 4-hour DR
- **Reward function** (at a given state S_t):
 - Single-objective version: deterministic, maximize REP profit $F_t(A_t)$
 - Bi-objective: stochastic, maximize REP profit $F_t(S_t, A_t)$ β is weight, minimize risk, $R_t(S_t, A_t)$ $(1-\beta)$ is weight
 - β in $[0,1]$



Conclusions for Dynamic Programming Study

- DP viable for **real-time** DR scheduling
- Savings of **\$10-\$25/ customer/ year**, or **10%- 25% additional savings possible on top of profit margins of \$100/ customer annually**
- Less risky to call 1-hour events in the evening
- Morning events are the riskiest
- Historically:
 - **1-hour events** would have generated the most savings
 - The **shoulder season** had the most potential savings
 - Few events generated the vast majority of potential savings



Highlighted Area #2

Supply Security in International Natural Gas Markets and the Effects of Expanding the Panama Canal on Liquefied Natural Gas (LNG) Flow Électricité de France (EDF)

- S. Moryadee, S.A. Gabriel, H.G. Avetisyan, 2014. "Investigating the Potential Effects of U.S. LNG Exports on Global Natural Gas Markets, 2(3-4), *Energy Strategy Reviews*, 273-288.
- S. Moryadee, S.A. Gabriel, F. Rehulka, 2014. "The Influence of the Panama Canal on Global Gas Trade *Journal of Natural Gas Science and Engineering*, 20, 161-174.
- S. Moryadee and S.A. Gabriel, 2017. "Panama Canal Expansion: Will Panama Canal be a Game Changer for LNG Exports to Asia?, *Journal of Energy Engineering*, 143(1), February.



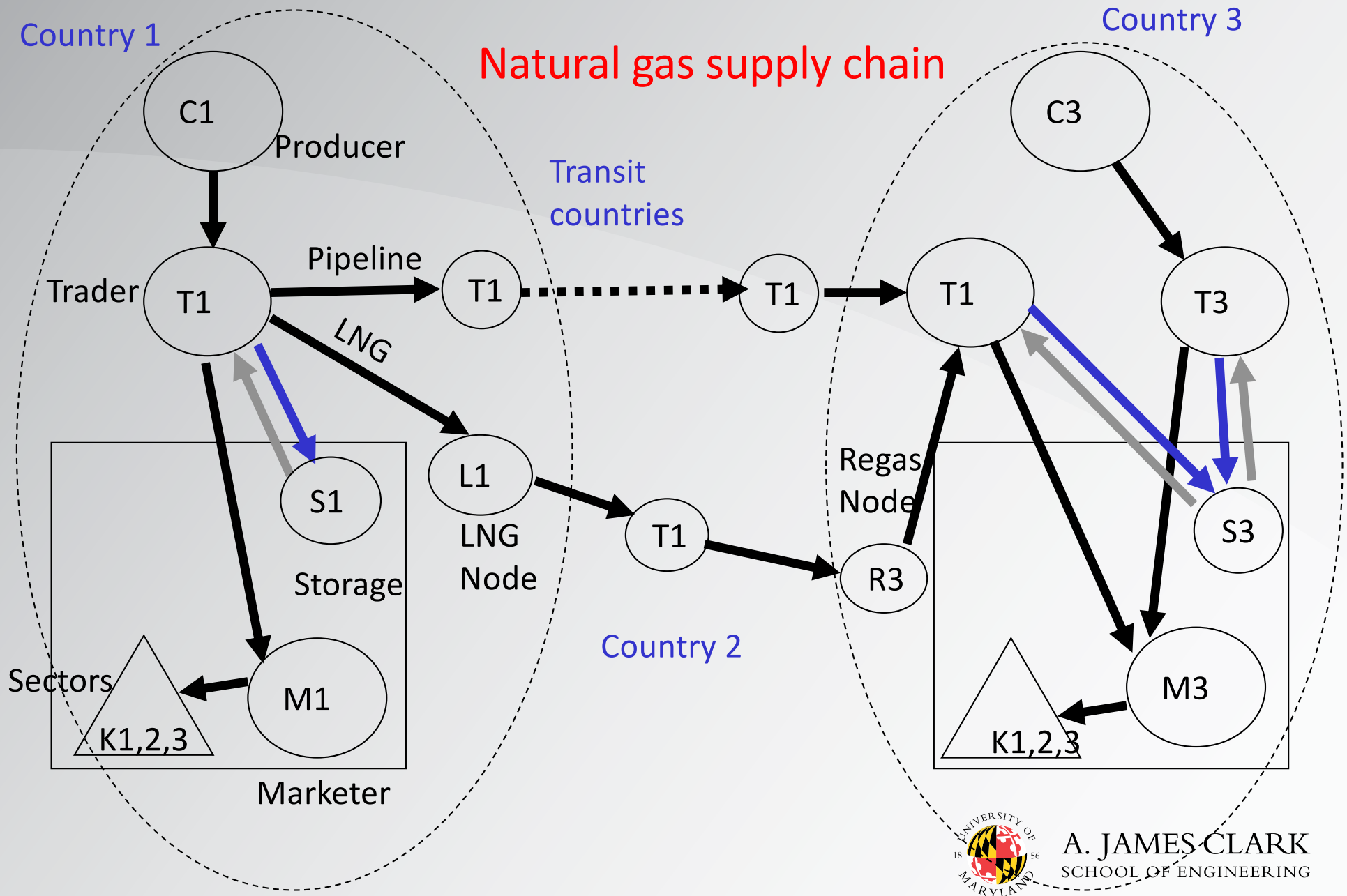
Supply Security in International Natural Gas Markets

Key Issues:

- European energy security issues (vis-à-vis Russian gas problems)
- How to achieve supply diversity including U.S. exports of LNG to Europe and Asia
- How do U.S. LNG exports influence worldwide gas flows with an expanded Panama Canal?
- World Gas Model developed and used to answer these questions (it's a large-scale, Nash-Cournot equilibrium model)



Representation of Gas Market in World Gas Model to Analyze LNG Issues and Panama Canal's Influence



The World Gas Model, A Large-Scale Nash-Cournot Market Equilibrium Model

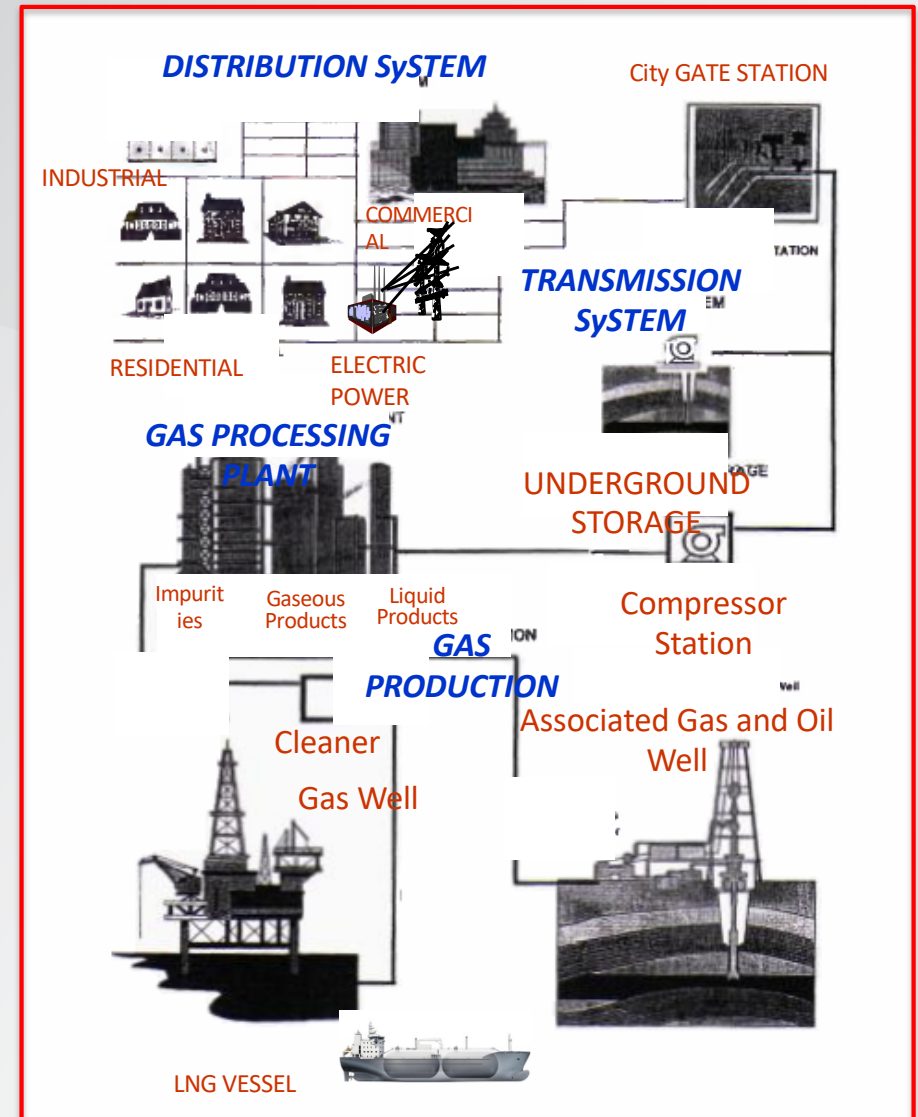
- Production/Consumption Nodes: 41 (Groups of countries, countries, regions)
- Covers over 95% of worldwide consumption
- 10 periods: 2005-2050, calibration year 2010
- Typical decision variables
 - Operating levels (e.g., production, storage injection)
 - Investment levels (e.g., pipeline, liquefaction capacity)
- Other
 - Market power aspects (traders)
 - LNG contracts database
 - Seasonality of demand: low and high demand
 - Environmental policy consideration: Carbon costs for supply chains
- Computational aspects
 - Large-scale complementarity problem (KKT optimality conditions for all players + market-clearing conditions)
 - ~78,000 vars. Solves in ~240 mins (8GB, 3.0 GHz)
 - MCPs are examples of non-convex problems (via the complementarity constraints)
 - Improved WGM, S. Moryadee Ph.D. thesis 2015



The World Gas Model

- The World Gas Model has been used by a number of governments such as:
- Norway
 - Research Council of Norway, 3-year project with NTNU, University of Maryland, Tsinghua University, SINTEF, Joint Global Change Research Institute
 - Statistics Norway
- France
 - Gaz de France (then GDF Suez, now Engie)
 - Electricité de France
- United States
 - U.S. Department of Energy

Natural gas supply chain



Current World Gas Model Configuration

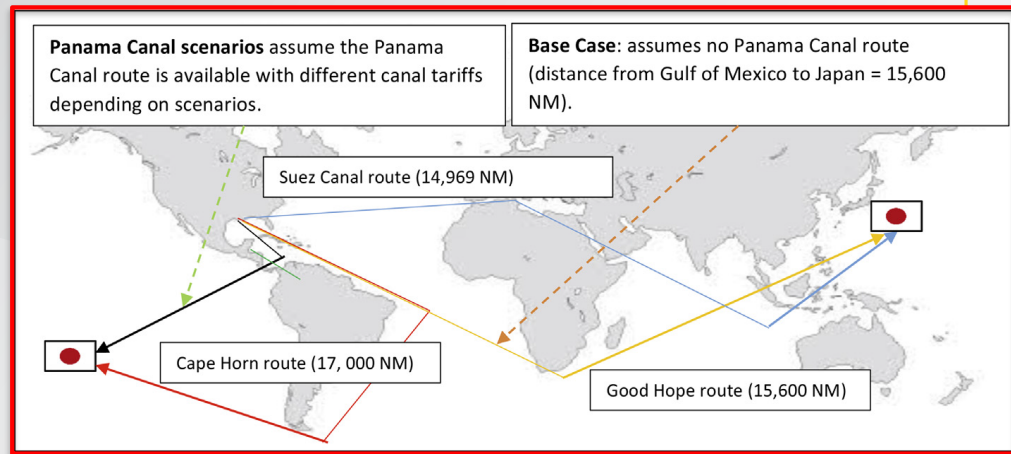
(Moryadee, 2015)

	WGM
Market players with separate optimization problems	Producers Traders Pipeline operator Storage operator Marketers Liquefier Regasifiers LNG shipping operator Canal operators
LNG shipping cost	Endogenous
Investment for producers	Endogenous
Investment for LNG tanker	Yes
Limitation on LNG shipping	Constraint on LNG Shipping operator
LNG routes	Flexible up to 3 routes
Number of variables	~ 110,000 vars



Much Shorter Distances for U.S. Gulf of Mexico LNG Exports to Asia via the Panama Canal

Origin	Via Panama	Via Suez	Around Cap Horn	Around Good Hope	Destination
Gulf of Mexico	3,733	21,637	9,783	19,713	Mexico West
	4,449	19,723	13,476	20,266	Chile
	9,756	14,449	17,060	15,697	Japan
	12,147	11,910	16,900	13,157	Singapore
Trinidad	3,331	20,272	7,643	17,573	Mexico West
	4,048	18,358	11,336	18,126	Chile
	9,355	13,054	14,920	13,557	Japan
	11,746	10,545	14,761	11,027	Singapore
Norway	7,471	19,474	10,801	19,601	Mexico West
	8,188	17,559	14,493	20,155	Chile
	13,494	12,285	18,078	15,585	Japan
	15,886	9,746	17,918	13,046	Singapore



Popils,2011

- Massive time saving on voyages to Japan, South Korea, Taiwan and China
- Avoid Cape Horn during winter season for potential deliveries to western coast of North and Central America
- Panama Canal expansion to be able to handle more and larger ships



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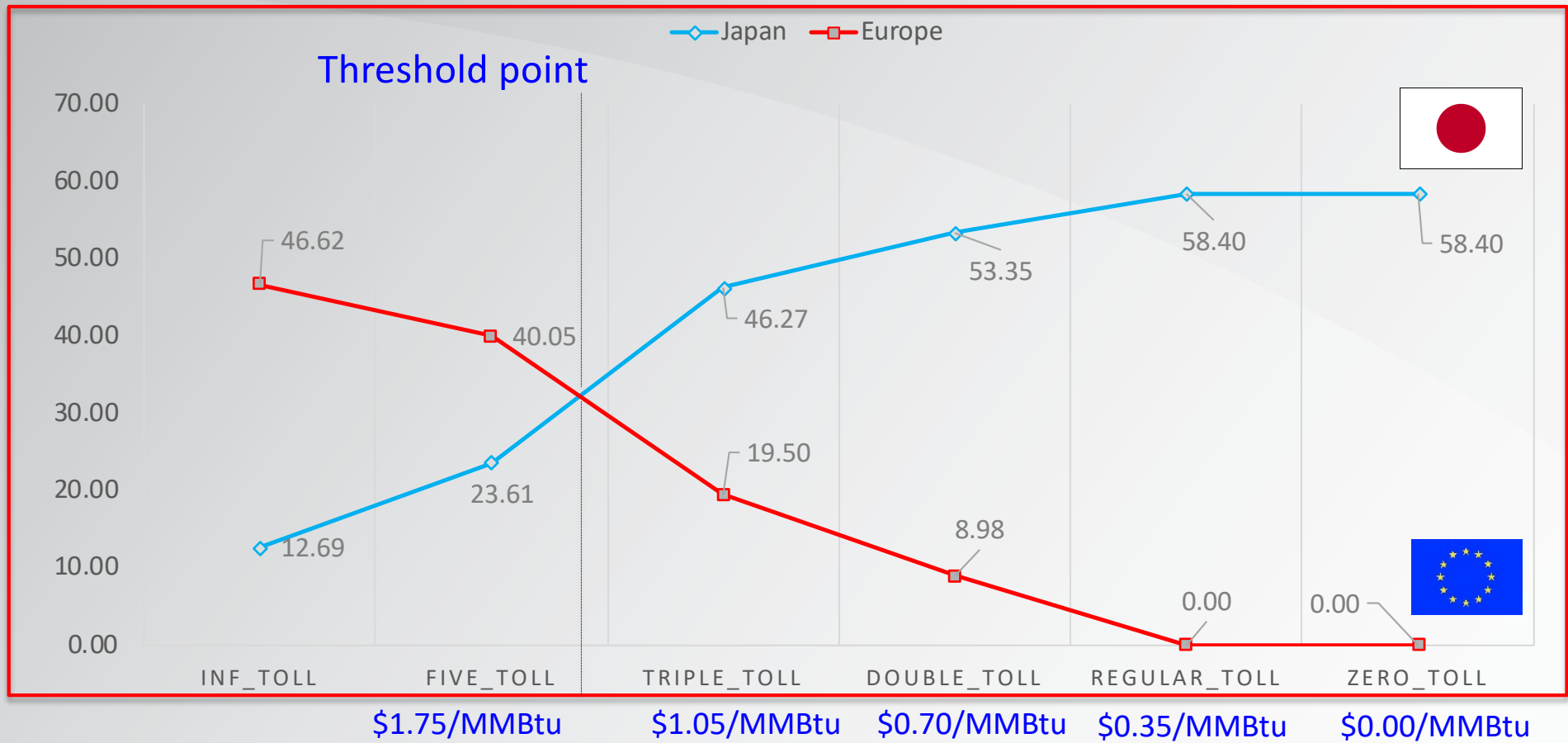
EDF-WGM Sensitivity Analysis Scenarios

Scenarios	Assumptions
Zero_Toll	"Zero Tariff" :tariff is \$0/trip or \$0.00/MMBtu
Regular_Toll	"Regular Tariff" : Canal Tariff tariff = \$/trip or \$0.35 /MMBtu
Double_Toll	"Double Tariff" :Canal tariff=Regular tariff X 2 = \$0.70 /MMBtu
Triple_Toll	"Triple Tariff" :Canal tariff=Regular tariff X 3 = \$1.05 /MMBtu
Fivefold_Toll	"Fivefold Tariff" :Canal tariff=Regular tariff X 5= \$1.75 /MMBtu
Inf_Toll	"Infinite Tariff" : Canal tariff= large number \$9,999/kcm



Impacts of Canal Tolls on Flows from U.S. Gulf of Mexico (US7 Node)

FLOWS FROM US7 TO EUROPE/ ASIA IN BCM/Y FOR 2035

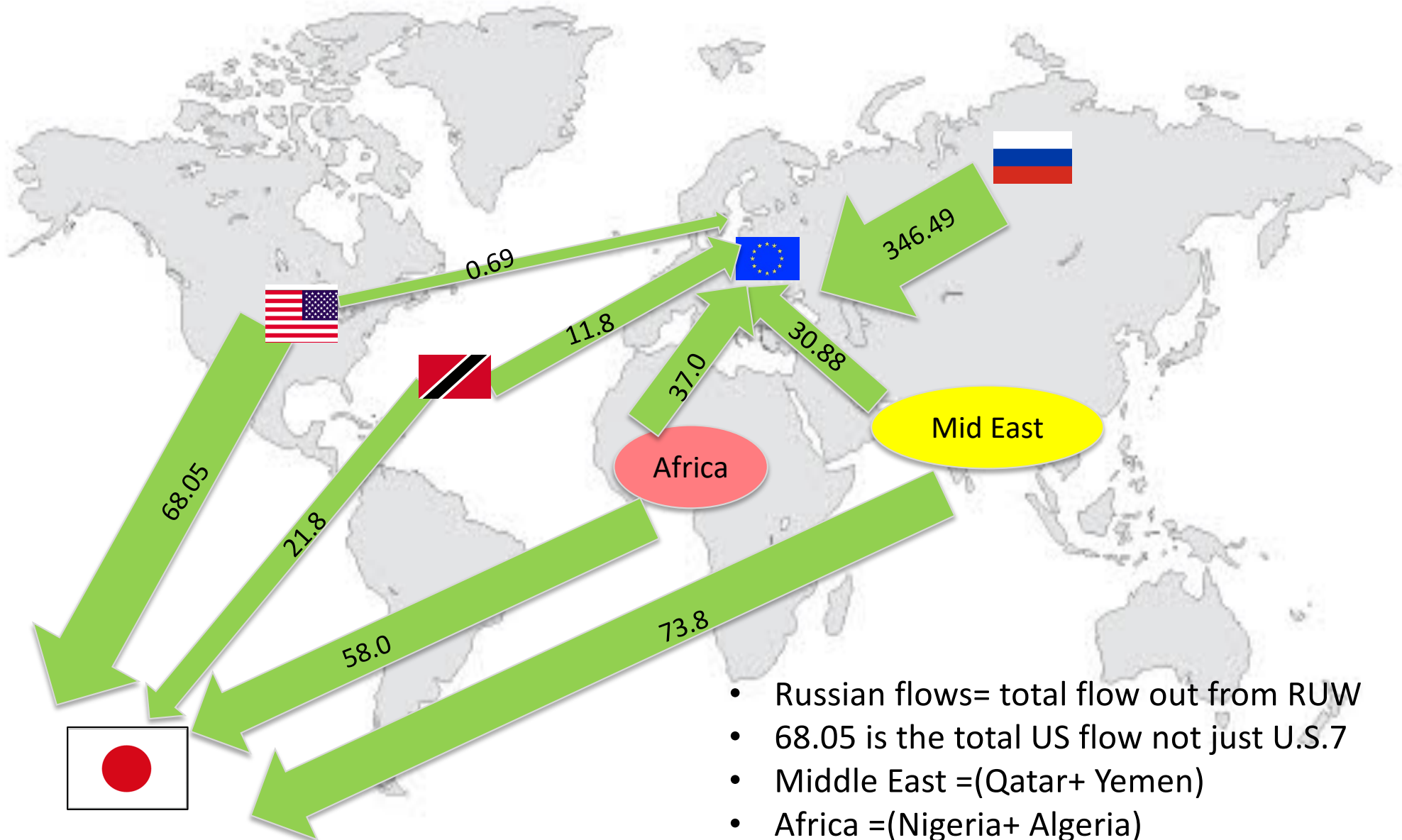


← Increasing toll

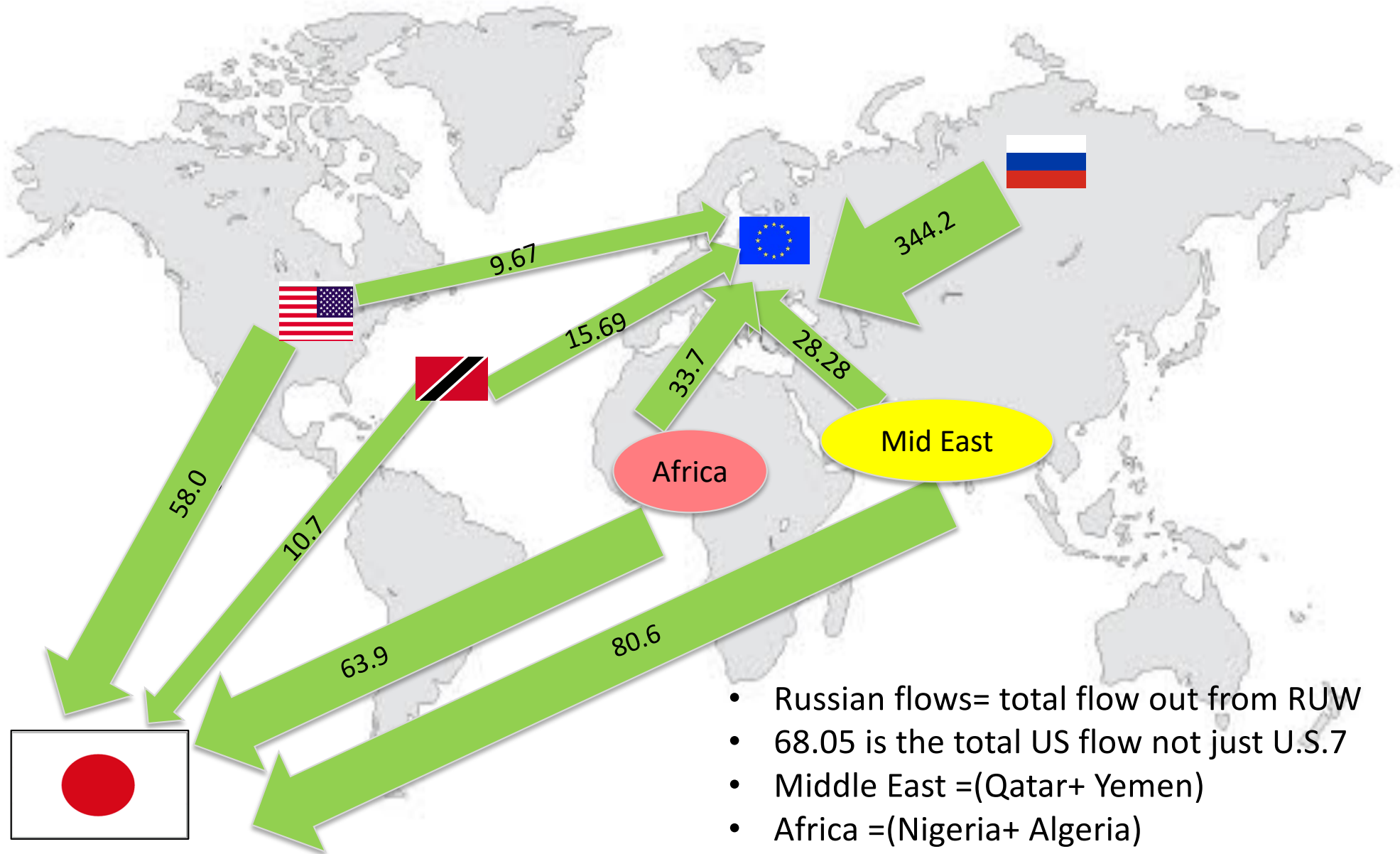


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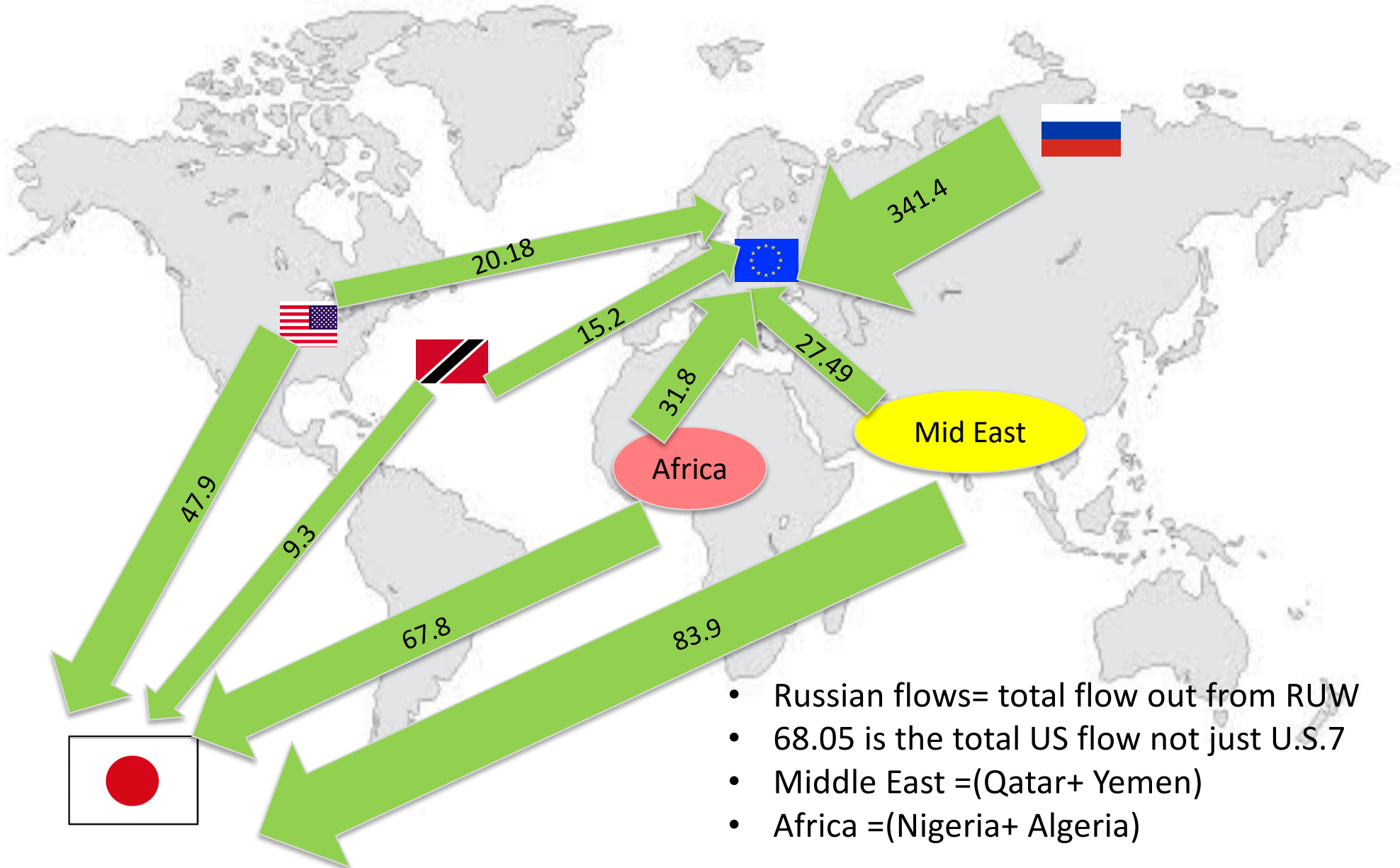
Dynamics of Flows: Regular Tariff Scenario, Flows in Bcm/y for 2035



Dynamics of Flows: Double Tariff Scenario, Flows in Bcm/y for 2035

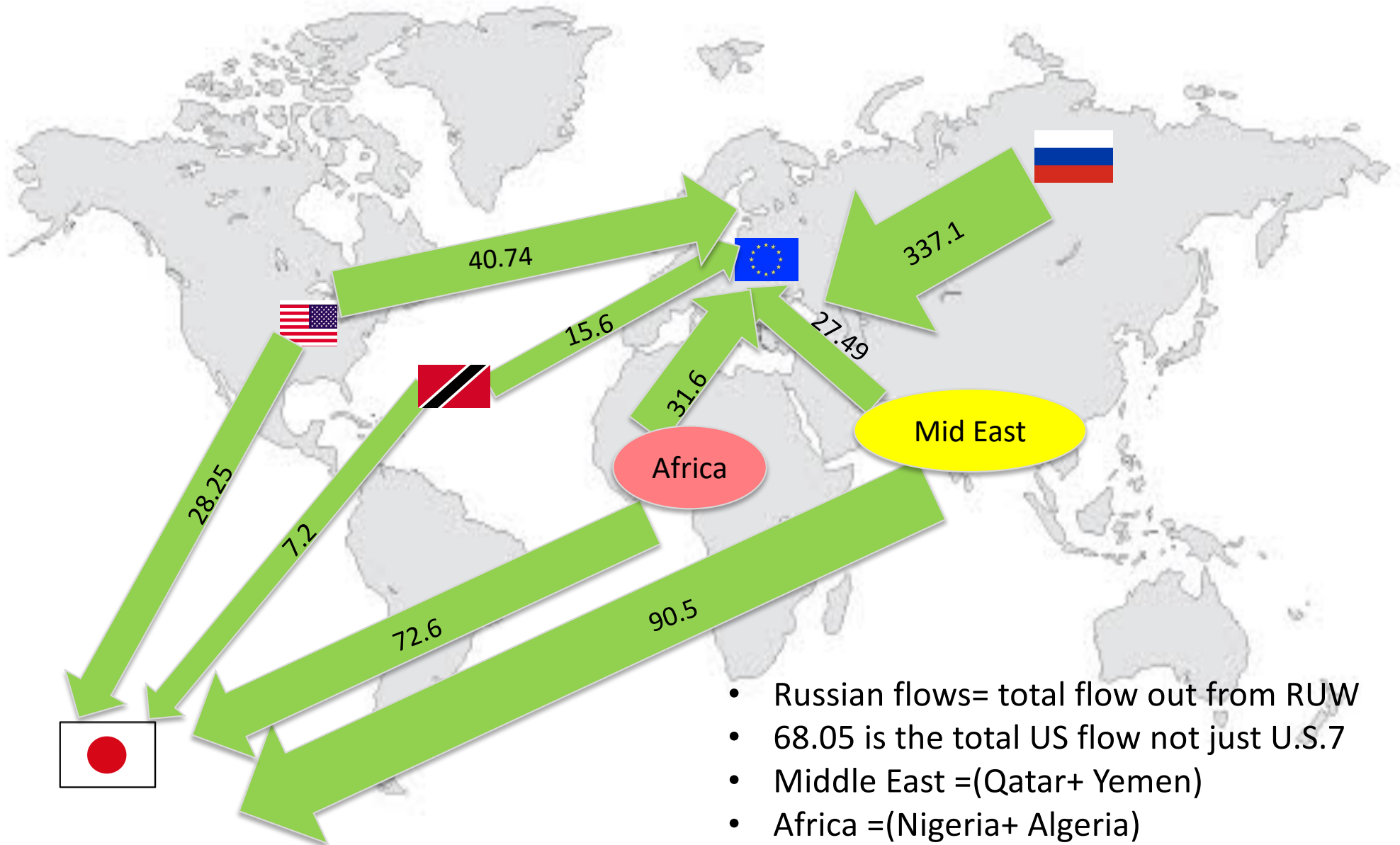


Dynamics of Flows: Triple Tariff Scenario, Flows in Bcm/y for 2035



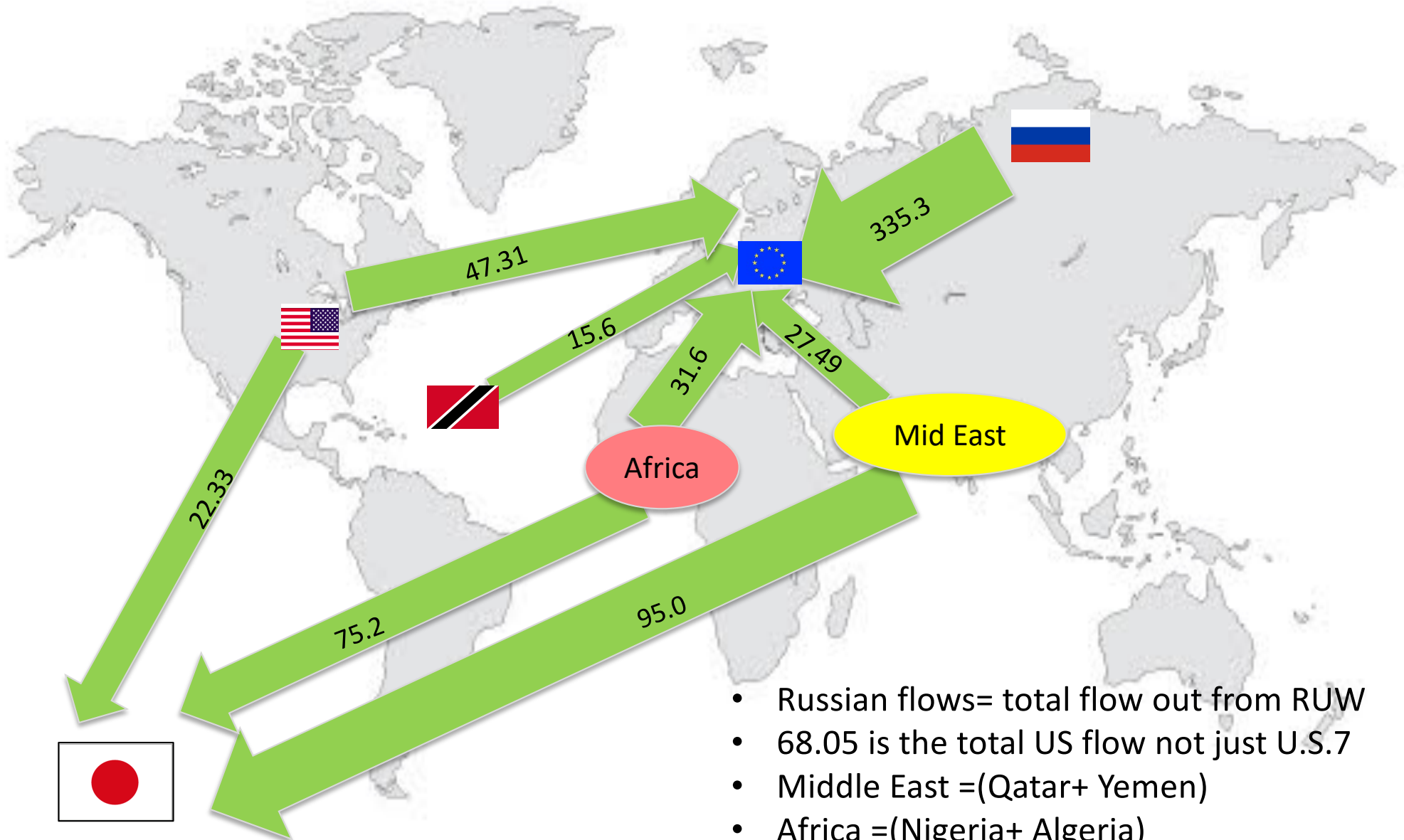
- Russian flows= total flow out from RUW
- 68.05 is the total US flow not just U.S.7
- Middle East =(Qatar+ Yemen)
- Africa =(Nigeria+ Algeria)

Dynamics of Flows: Five-fold Tariff Scenario, Flows in Bcm/y for 2035



- Russian flows= total flow out from RUW
- 68.05 is the total US flow not just U.S.7
- Middle East =(Qatar+ Yemen)
- Africa =(Nigeria+ Algeria)

Dynamics of Flows: Infinite Tariff Scenario, Flows in Bcm/y for 2035



Four Energy Projects Using a Bilevel Optimization/Mathematical Program with Equilibrium Constraints (MPEC) Approach

- #1. DC Water and Sewer Authority, Wastewater-to-Energy
(S.A. Gabriel, C. U-tapao, S. Moryadee)
- #2. Agency-level Energy Conservation with Shared Savings
(B. Champion, S.A. Gabriel)
- #3. Power Market Investments with Endogenous Prices,
Capacities and Quantities
(H. Bylling, S.A. Gabriel, T. Boomsma)
- #4. Power Market Investments with Rolling-Horizon,
Endogenous Probabilities
(T. Kallabis, S.A. Gabriel, C. Weber, in preparation)



MPEC Formulation

$$\begin{aligned} \min f(x, y) \\ \text{s.t. } (x, y) \in \Omega \\ y \in S(x) \end{aligned}$$

where

Ω set of constraints for (x, y)

$x \in R^{n_x}$ upper-level variables

$y \in R^{n_y}$ lower-level variables

$f(x, y)$ upper-level objective function

$S(x)$ solution set of lower-level problem (opt. or game)

Note: Stochasticity can enter in three places:

1. Top-level objective
2. Top-level feasible region
3. Bottom-level problem

-Could be chance constraints, recourse problem, conditional value-at-risk, etc.



MPEC Formulation

- Many problems in engineering and economics can be put into this format; some salient examples:

- Stackelberg leader-follower problems

- Upper-level problem: leader

- Lower-level problem: followers

- Other problems are of the form:

- Upper-level problem: investments

- Lower-level problem: operations/markets

- Equilibrium network design

- Origin-destination demand adjustment problem (model calibration + MCP/VI)



#1, Wastewater-to-Energy, A Two-Level

Key Issues:

Optimization

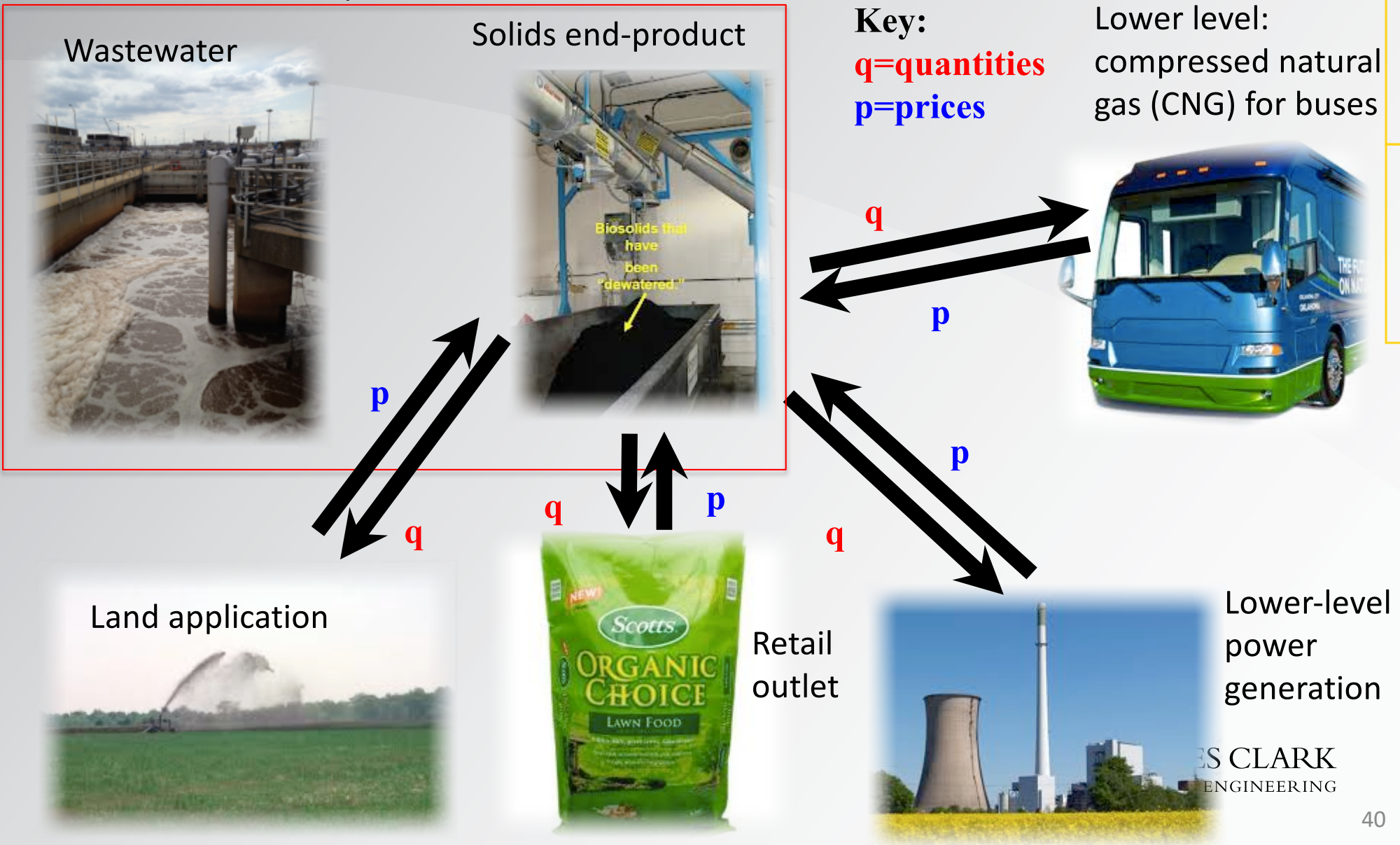
- How best to use wastewater to produce:, renewable electric power, Compressed natural gas (for Washington, DC buses, fleets), high-end fertilizer for farms, Class B biosolids, residential natural gas sector
 - Developed a stochastic, two-level optimization model to answer the above questions, recourse problem at the top level, stochasticity due to scenario tree for energy prices, amount of wastewater, etc.
 - Hard non-convex problems to solve, especially when they are large-scale as in this application
- C. U-tapao, S.A. Gabriel, C. Peot, and M. Ramirez, 2015. "A Stochastic, Multiobjective, Mixed-Integer Optimization Model for Management of Wastewater-Derived Energy," *J. of Energy Engineering*, 141(1).
 - C. U-tapao, S. Moryadee, S.A. Gabriel, C. Peot and M. Ramirez, 2016. "A Stochastic, Two-Level Optimization Model for Compressed Natural Gas Infrastructure Investments in Wastewater Management," *Journal of Natural Gas Science & Engineering* 28, 226–24 .



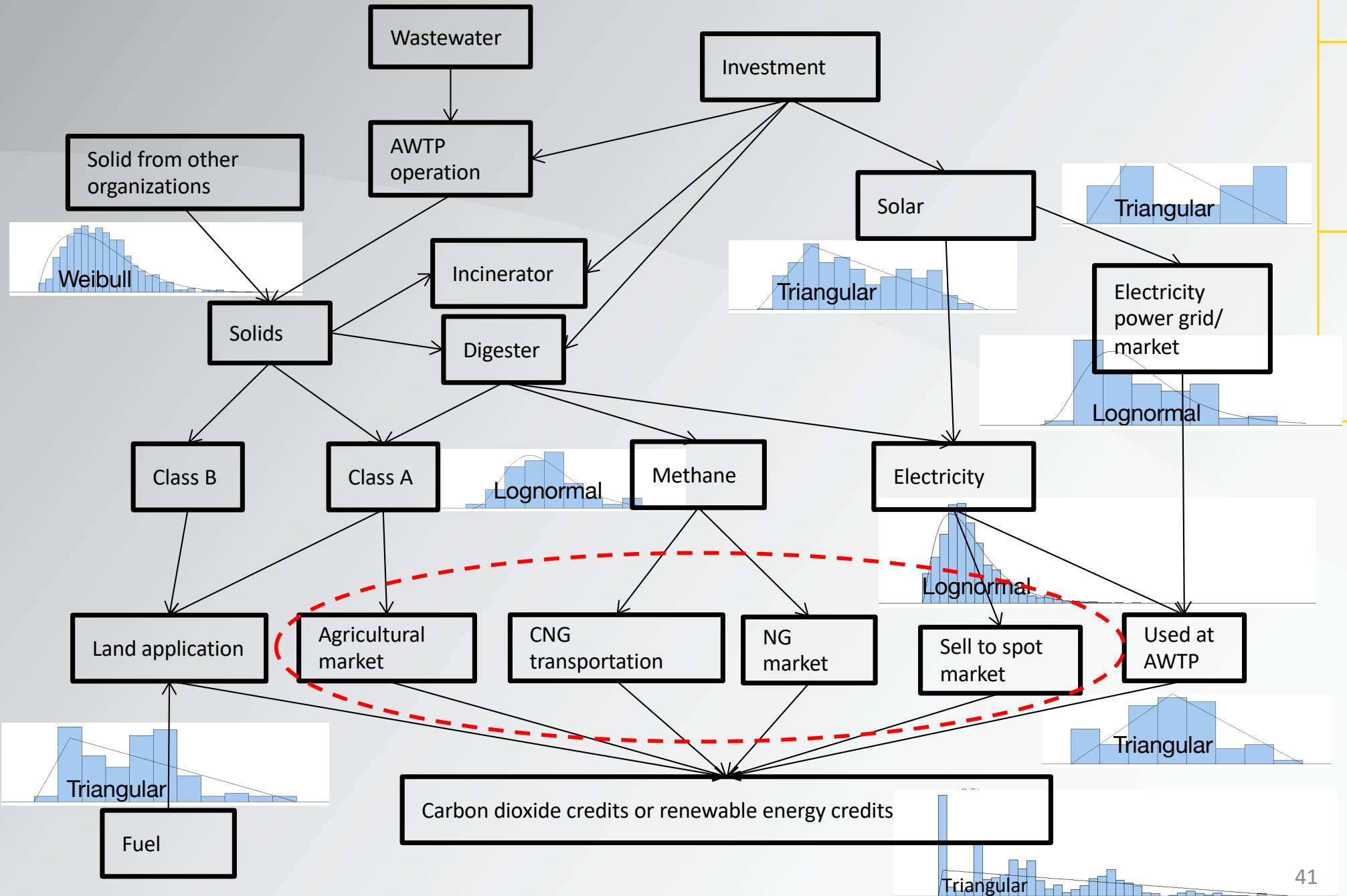
Overall Stochastic MPEC Formulation

DC Water and Sewer Authority

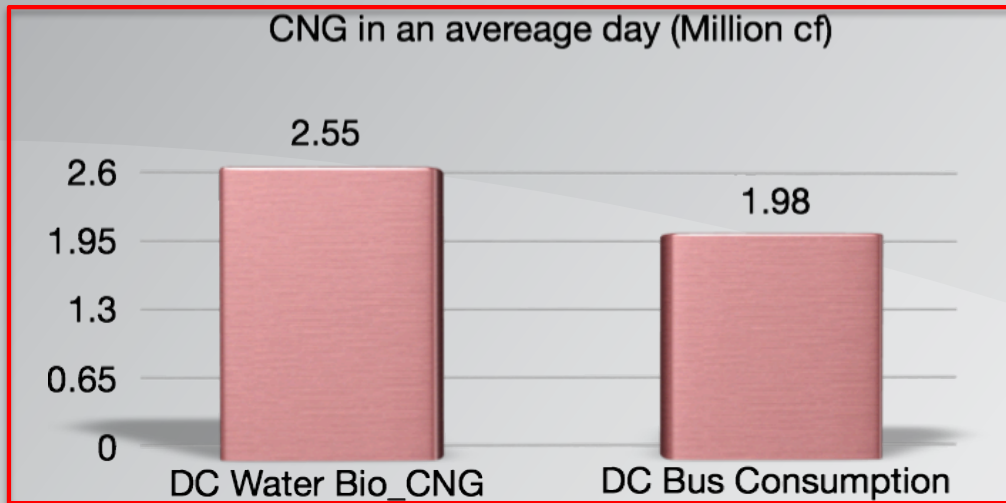
Top level: Stochastic optimization problem with recourse for wastewater treatment plant



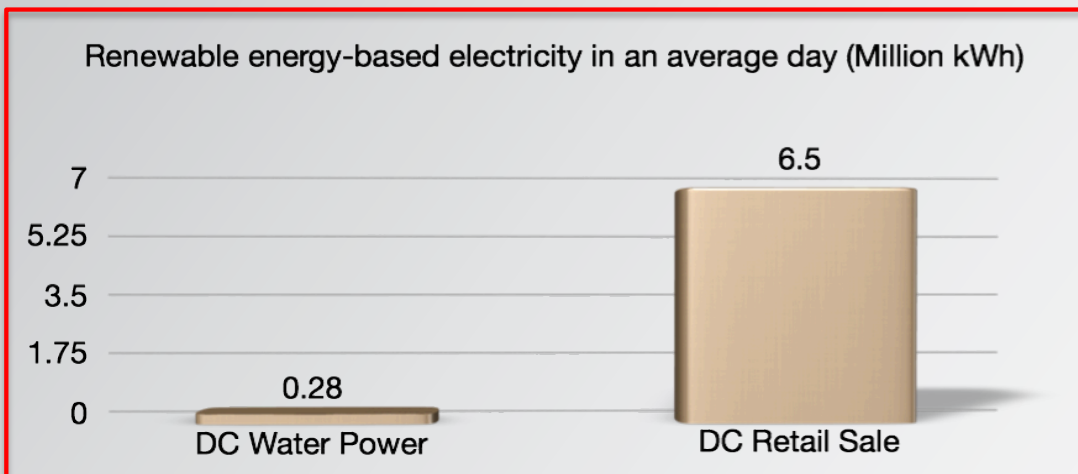
Process Diagram of the Stochastic MPEC



DC Water's Bio-CNG and Electricity in Relevant Markets



- CNG and Power
- Bio_CNG production from DC Water is enough for DC Bus consumption.
- DC Water CNG station may be an option to support CNG vehicle (DC has 0 public, 2 private, MD has 3 public, 2 private CNG station).
- Electricity generated from DC Water can supply renewable-based electricity to the retail sale company (Pepco).
- Electric power from DC Water is 10 MW, Pepco doesn't want to lose it.



#2, Agency-Level Energy Conservation

- B. R. Champion and S.A. Gabriel, 2015. " An improved strategic decision-making model for energy conservation measures," *Energy Strategy Reviews* 6, 92-108.
- B.R. Champion and S.A. Gabriel, 2017. "A Multistage Stochastic Energy Model with Endogenous Probabilities and a Rolling Horizon," *Energy and Buildings*, 135, 338-349. B. R.
- Champion and S.A. Gabriel, Risk-based Multistage Stochastic Energy Conservation Project Selection, October 2016 (in review).

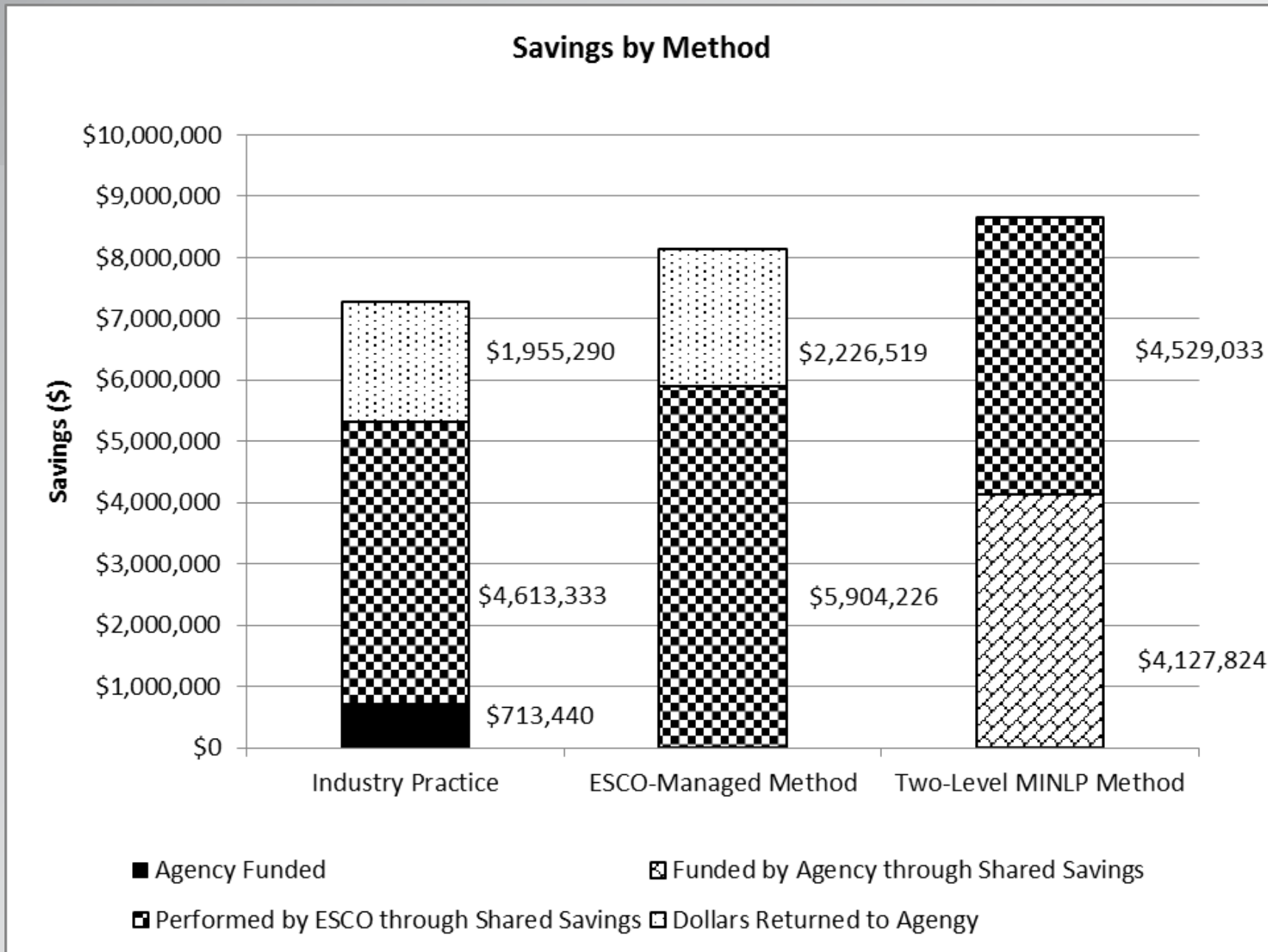


Agency-Level Energy Conservation

- An energy conservation program is required to meet the deadlines of federal regulation
- 48 projects have been identified and are required to meet the savings goal of the federal regulation
- Each project has an investment and a annual savings (energy saved * energy rate)
- Energy project budget is fixed
- Agency (top-level player) to determine which projects to do by itself, which ones to contract out to ESCOs (bottom-level players)
- **Problem: What is the minimum cost to complete all projects?**
- **What is the smallest capital request that can be made to complete all projects?**
- **How can savings be used to fund future projects with savings, understanding that energy rates will change?**
- **How can it get supplemental help executing projects when these costs are too high?**



Agency-Level Energy Conservation



#3, Power Market Investments with Endogenous Prices, Capacities and Quantities

- H. Bylling, S.A. Gabriel, T.K. Boomsma, 2019. "A Parametric Programming Approach to Bilevel Optimisation with Lower-Level Variables in the Upper Level," *Journal of the Operational Research Society*, May, 2019.
- H.C. Bylling, Trine K. Boomsma, S.A. Gabriel, "A Parametric Programming Approach to Bilevel Electricity Transmission Investment Problems," Chapter 6 in *Transmission Network Investment in Liberalized Power Markets*, Springer (Lecture Notes in Energy), Editors: M. R. Hesamzadeh (KTH, Sweden), J. Rosellon (CIDE, Mexico and DIW, Germany), I. Vogelsang (Boston University, US), accepted June, 2019.



Power Market Investments with Endogenous Prices, Capacities and Quantities

Bilevel Investment Problem

$$\begin{aligned} \min c^T x + g(y, \lambda) \\ Ax = b \end{aligned}$$

Investment decision: upper-level variable (x)

$$\begin{aligned} \min p_1^T y_1 \\ y_1 \end{aligned}$$

$$C_1 y_1 = D_1 x + e_1 (\lambda_1)$$

$$\begin{aligned} \min p_2^T y_2 \\ y_2 \end{aligned}$$

$$C_2 y_2 = D_2 x + e_2 (\lambda_2)$$

$$\begin{aligned} \min p_3^T y_3 \\ y_3 \end{aligned}$$

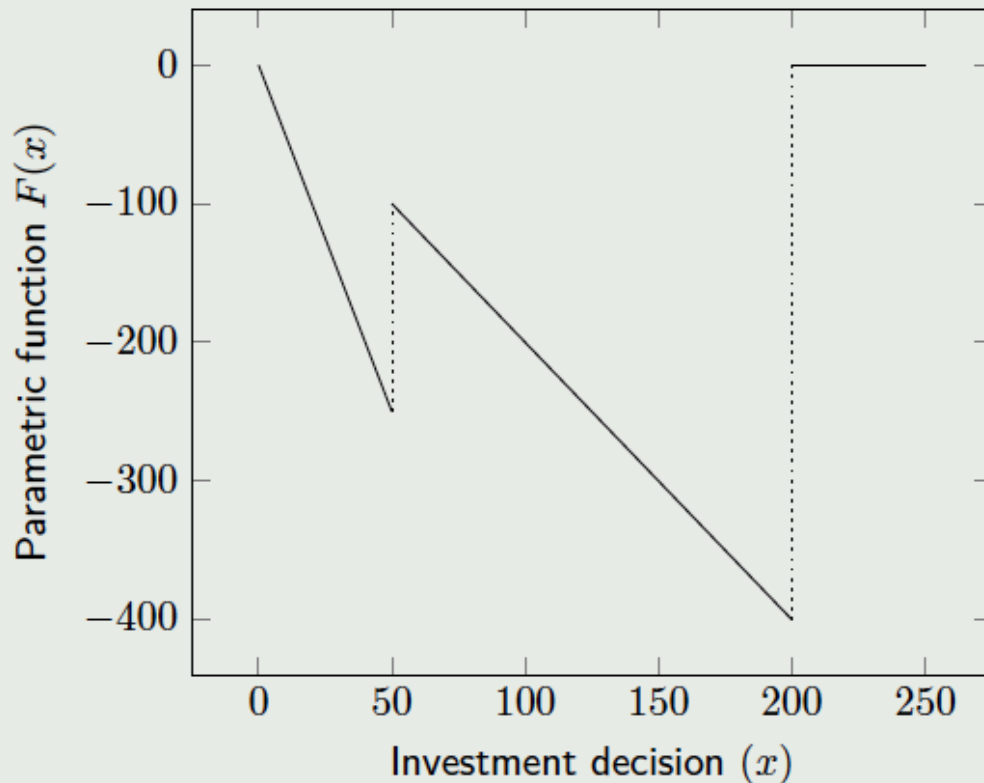
$$C_3 y_3 = D_3 x + e_3 (\lambda_3)$$

Market clearings: lower-level primal (y) and dual (λ) variables.



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Investment Example



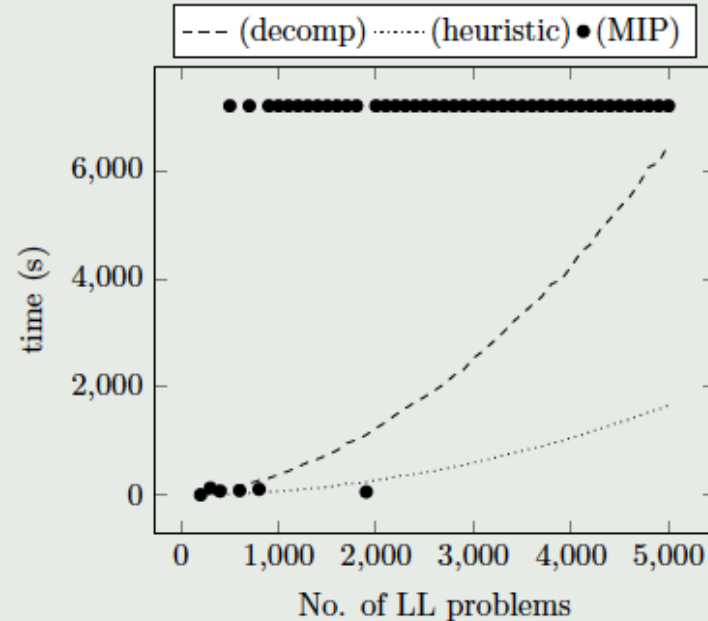
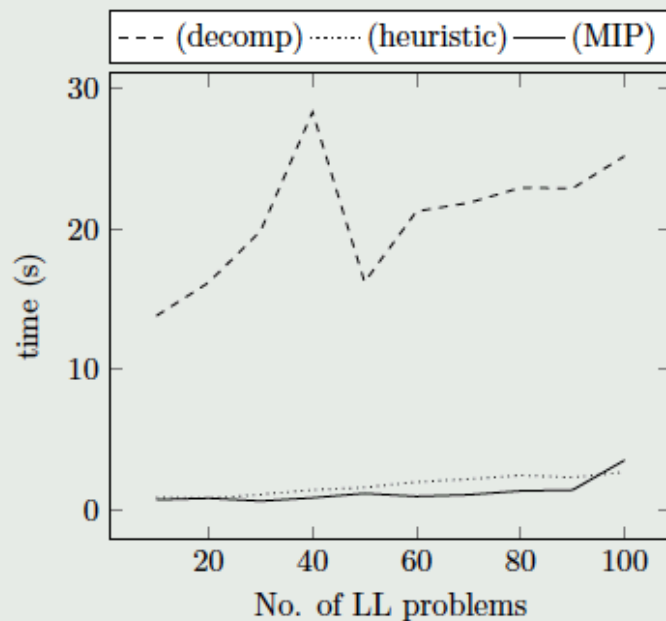
Example:

- 3 existing units in power market.
- x invested capacity (MW)
- $F(x)$ profit from new unit (EUR).



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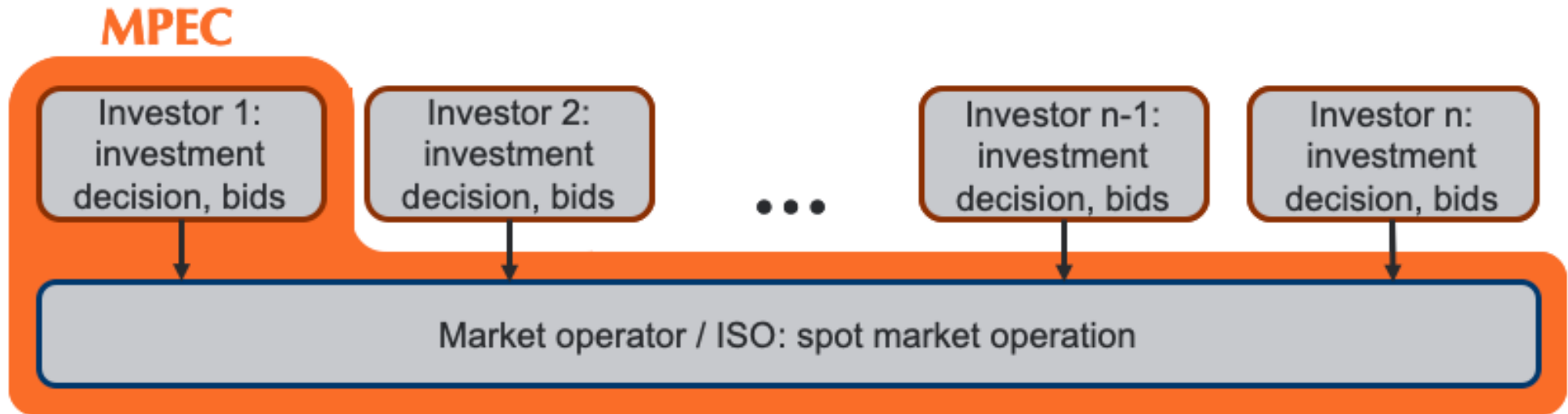
Solution time



(a) No of LL problems from 10 to 100. (b) No of LL problems from 200 to 5000.

Figure: Solution times for increasing number of LL problems.

#4, Power Market Investments with Rolling-Horizon, Endogenous Probabilities



- Goal: improve existing investment MPEC modeling
- Implement dynamic multi-period structure and a rolling planning horizon
- Capture value of rolling horizon, tradeoff between computation time and solution quality
- Allows creating more realistic model configurations
- Capture optionality value of multi-stage investment process
- Contribution to strategic generation investment literature through rolling horizon and link to real option valuation of power plant investments
- Stochastic demand



Power Market Investments with Rolling-Horizon, Endogenous Probabilities

- Model configurations defined by included model features
 - Perfect foresight (deterministic), rolling horizon, recourse action (abandon projects)

Configuration	Perfect foresight	Rolling horizon	Recourse Action
Perfect Foresight (PF)	✓	X	✓
Stochastic Recourse Action (RA)	X	X	✓
Stochastic Optimization (ST)	X	X	X
Rolling Horizon (RH)	X	✓	✓
Limited Planning (LP)	X	✓	X

- Perfect foresight (PF) configuration as benchmark
- Impact of rolling planning observable: RA vs. RH and ST vs. LP
- Impact of investment stage structure / recourse action:

Power Market Investments with Rolling-Horizon, Endogenous Probabilities

- All model configuration still show similar average profits
 - Negative effect of rolling horizon planning horizon not as pronounced
- Computation time increased massively
 - Fully stochastic RA configuration does not terminate at 0% MIP gap
 - Variations of MIP gap show tradeoff between computation time and MIP gap

Config.	Profit [m. EUR]	Delta to PF [m. EUR]	Base cap. [MW]	Peak cap. [MW]	Comp. time [seconds]	MIP gap
PF	5,284.7	–	1,500	856	22.8	0.00%
RH	5,273.2	–11.5	1,500	947	35.3	10 ⁻⁶
LP	5,267.0	–17.7	1,500	1116	27.2	10 ⁻⁶
RA _{10%}	5,273.3	–11.4	1,500	961	28.9	10.00%
RA _{7.5%}	5,274.5	–10.2	1,500	961	1430.7	7.50%
RA _{5%}	–	–	–	–	> 21600	5.00%

Thank you and any questions?



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